• **Input impedance** (well above the –3dB point, so that \(C_1\) is not relevant).

\[
\frac{1}{Z_{\text{in}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{\beta R_E} \quad \text{\(Z_{\text{in}} \approx 10k\Omega\)}
\]

• **Output impedance**

\(Z_{\text{out}} = R_C\)

Remember, the impedance looking into the collector is so large compared with \(R_C\) that it can be neglected as a parallel contribution.

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The input impedance is rather poor, because of the bias circuit. So if the source impedance is $1\,\text{k}\Omega$, for example, we lose about 10\% of our gain.

We will soon learn how to “bootstrap” the bias circuit to greatly increase the input impedance.
Measuring $Z_{\text{in}}$ and $Z_{\text{out}}$

- *Think in terms of simple voltage dividers.*
- *This also highlights the importance of understanding these two quantities!*

\[ V = \frac{R_L}{R_L + Z_{\text{out}}} \cdot V_S \]

\[ V = \frac{Z_{\text{in}}}{R_S + Z_{\text{in}}} \cdot V_S \]
Grounded-Emitter Amp

• V gain of ~260, but this circuit is a disaster waiting to happen! You can make it work in PSpice by tuning the biasing, but in real life
  – The bias point will not be stable with temperature or changing parts!
  – Also, the linearity will be poor, except with very small input signals (<<10mV input).
  – Input impedance of the transistor base is low ($\beta r_E$) and unstable.

• By adding an emitter resistor we introduce “negative feedback.” At the cost of reduced overall voltage gain, we greatly improve linearity and stability.
**Linearity with and without Negative Feedback**

With 1k emitter resistor.

400 mV sine wave input

Without emitter resistor.

19 mV sine wave input
Common-Emitter Amp with High Gain

- Use of the bypass capacitor around the emitter resistor ensures that the biasing is stable, even with large gain.
- High gain is achieved above 100 Hz. Below 100 Hz we don’t care.
- But still, the linearity is poor, except with very small input signals, and the input impedance of the transistor base is still low.
More technical details on the performance of

BIPOLAR TRANSISTORS
Ebers-Moll Equation

- Simply the I/V relationship for a forward-biased PN junction (base-emitter).
- But remember that ~99% of the current flows to the collector.

\[ I_C = I_S \left( e^{V_{BE}/V_T} - 1 \right) \]

- \( I_C \) increases with \( V_{BE} \) by about 17 mV per octave (or ~60 mV per decade).  
\[ e^{17/25} \approx 2 \]

\[ V_T = kT/e \approx 25 \text{mV} \]

\[ V_{BE} = \frac{kT}{q} \ln \left( \frac{I_C}{I_S} + 1 \right) \]

\[ I_B = \frac{I_C}{h_{FE}} \]

\[ I_C \text{ current scale} \]

Figure 2.32. Transistor base and collector currents as functions of base-to-emitter voltage \( V_{BE} \).
Temperature Dependence

\[ I_C = I_S \cdot \left( e^{V_{BE} / kT} - 1 \right) \]

Remember: this equation does not show all the T dependence, as \( I_S \) increases very rapidly with temperature.

\( I_C \) grows at about 9% per degree C if \( V_{BE} \) is held constant.

or equivalently

\( V_{BE} \) falls at about 2 mV per degree C if \( I_C \) is held constant.

This can cause serious problems, so designs should cope with this by

- Negative feedback (e.g. using an emitter resistor)
- and/or compensation (design tricks)
Emitter Resistance

- The emitter has a dynamic impedance \( r_E \) given by the Ebers-Moll equation.
- The voltage division between \( r_E \) and the emitter resistor of the emitter-follower circuit results in a less-than-unity gain for an emitter follower:

\[
G = \frac{R_E}{R_E + r_E}
\]

\[
r_E = \frac{\Delta V_{BE}}{\Delta I_E} \approx \frac{\Delta V_{BE}}{\Delta I_C} = \frac{kT/e}{I_C} = \frac{25\text{mV}}{I_C}
\]
Current Gain ($\beta$) Changes with $I_C$

- **2N3904 NPN Transistor**
- $V_{CE} = 10\,\text{V}$
- $f = 1.0\,\text{kHz}$
- $T_A = 25^\circ\text{C}$

*Typical response, from the data sheet*
Early Effect

Larger reverse bias on the Base-Collector junction reduces the thickness of the Base region that charge must diffuse through to get from emitter to collector.

Therefore one gets more current for the same $V_{BE}$.

Or equivalently, for a fixed collector current, less Base-Emitter bias voltage is needed:

$$\Delta V_{BE} = -\alpha \Delta V_{CE}$$

$$\alpha \approx 0.0001$$

Since this means that the collector current depends on the collector voltage, then the collector output impedance is affected!
Early Effect and Collector Output Impedance

\[ \Delta V_{BE} = -\alpha \Delta V_{CE} \]

\[ \alpha \approx 0.0001 \]

We normally operate the transistor in the pink region!

Two different values of \( V_{BE} \)

- Each curve corresponds to a different fixed \( V_{BE} \).
- The slope in the operating region is the inverse of the collector output impedance.

\[ Z_C = \frac{\Delta V_{CE}}{\Delta I_C} \approx \frac{1}{\alpha} \cdot \frac{\Delta V_{BE}}{\Delta I_C} = \frac{25\text{mV}}{\alpha \cdot I_C} = \frac{r_E}{\alpha} \]

e.g., for a 1 mA bias current, \( Z_C = 250 \text{k}\Omega \) (negative feedback from an emitter resistor can greatly increase this!)

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2N3904 NPN Transistor

Deriving the output impedance from the transistor data sheet:

\[ Z = \frac{1}{8.5 \times 10^{-6}} = 117 \text{k}\Omega \]

at 1 mA

\[ r_E = \frac{25}{1} = 25\Omega \]

\[ Z \approx \frac{25}{0.0001} = 250\text{k}\Omega \]

(Hence this estimate of \( \alpha = 0.0001 \) is probably low by a factor of 2 or so.)
Effect of $R_E$ on Current Source

**No emitter resistor.**

$Z_{out} = 170k\Omega$

**With emitter resistor.**

$Z_{out} = 1.5M\Omega$

The emitter resistor adds “negative feedback”. If the current from the emitter tries to increase, then $V_E$ increases, reducing $V_{BE}$ and thus reducing the current.

We saw that this same negative feedback stabilizes the CE amplifier.
Transistor Rules

• What you need to know to design circuits with bipolar transistors:
  – $V_{CE} > \sim 0.6V$ for operation in the “linear” region
    • $V_{CE}$ can go lower (e.g. $< 0.2V$), but then the transistor is getting into the saturation region where both junctions are forward biased and the transistor acts like a fully closed switch (see next slide).
  – $V_{BE} \sim 0.6$ to 0.7 V but varies with $I_C$ at a rate of 17 mV/octave
  – $I_C = \beta I_B$, but $\beta \sim 100$ varies with $I_C$, with $T$, and (slightly) with $V_{CE}$, and it also varies from one part to another! Do NOT rely on a precise value!
  – The emitter has an intrinsic impedance of $r_e \sim 25$ mV/$I_C$
  – For constant $I_C$, $V_{BE}$ decreases with $T$ at a rate of 2.1 mV/$^\circ$C
    • In general, use negative feedback to avoid any critical dependence on a precise value of $\beta$ and temperature!
  – For constant $I_C$, $V_{BE}$ varies slightly with $V_{CE}$ (Early effect)
    \[
    \Delta V_{BE} = -\alpha V_{CE} \quad \text{where} \quad \alpha \approx 0.0001
    \]
Transistor Switch & Saturation

Ideally \( V_{CE} \) goes to zero and we waste no power in the transistor!
Some More Transistor

CIRCUIT BUILDING BLOCKS
Simple Current Source

\[ Z_{out} = \frac{9.013 - 5.073}{(987.4 - 985.5) \times 10^{-6}} = 2.1 \, \text{M}\Omega \]

<table>
<thead>
<tr>
<th>RL</th>
<th>VC</th>
<th>IC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1k</td>
<td>9.013 V</td>
<td>987.4 (\mu)A</td>
</tr>
<tr>
<td>5k</td>
<td>5.073 V</td>
<td>985.5 (\mu)A</td>
</tr>
</tbody>
</table>
The cascode is a very common technique!

$$Z_{\text{out}} = \frac{9.031 - 5.159}{0.7 \times 10^{-6}} = 5.5 \, \text{M} \Omega$$

Q2 acts to hold the collector of Q1 at almost a constant voltage, to eliminate most of the Early effect.

The improvement would be far greater if RE were not present. The negative feedback produced by RE already makes quite a good current source!
Current Mirror Current Source

• Very commonly used, especially in integrated circuits.
• Matched pairs of transistors operated at identical temperature should be used; i.e. use pairs on a single IC chip.

\[ Z_{out} = \frac{11.36 - 6.86}{(1.372 - 1.136) \times 10^{-3}} = 19 \, k\Omega \]

Note how the two transistors always have exactly the same \( V_{BE} \)
Improved Current Mirror

- As usual, adding emitter resistors greatly improves performance.

<table>
<thead>
<tr>
<th>RL</th>
<th>VC</th>
<th>IC</th>
</tr>
</thead>
<tbody>
<tr>
<td>10k</td>
<td>9.879 V</td>
<td>987.9 µA</td>
</tr>
<tr>
<td>5k</td>
<td>4.973 V</td>
<td>994.6 µA</td>
</tr>
</tbody>
</table>

\[
Z_{out} = \frac{9.879 - 4.973}{(994.6 - 987.9) \times 10^{-6}} = 0.7 \, \text{MΩ}
\]
Wilson Current Mirror

- Q1 and Q2 should be matched
- Q3 keeps $V_{CE}$ constant for Q2 (and $V_{CE}$ is anyway constant for Q1). This is very similar to the cascode concept.
- Note how superbly it works even without emitter resistors.

### Table

<table>
<thead>
<tr>
<th>RL</th>
<th>VC</th>
<th>IC</th>
</tr>
</thead>
<tbody>
<tr>
<td>10k</td>
<td>9.517V</td>
<td>951.7μA</td>
</tr>
<tr>
<td>5k</td>
<td>4.769V</td>
<td>953.7μA</td>
</tr>
</tbody>
</table>

\[
Z_{out} = \frac{9.517 - 4.769}{2 \times 10^{-6}} = 2.4 \text{ MΩ}
\]
Wilson Current Mirror

Collector Voltage

Collector Current

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