Physics 5B
Lecture 21, March 5, 2012

Chapter 34, 2-Slit Interference
Two-Slit Experiment

The math is simple if the distance from the slits to the screen is very large compared with the slit spacing (Fraunhofer diffraction).

\[ d \sin \theta = m\lambda \]

- **Constructive**
  - \[ d \sin \theta = m\lambda \]

- **Destructive**
  - \[ d \sin \theta = (m + \frac{1}{2})\lambda \]
The position of this dark band depends on the slit width. We will study this in detail in Chapter 35.

\[ d \sin \theta = m\lambda \]

\[ d \sin \theta = (m + \frac{1}{2})\lambda \]
Electromagnetic Waves

Light is an **electromagnetic** transverse wave, with perpendicular oscillating electric and magnetic fields. We can take the amplitude to be the magnitude of the electric field $E$ (more on this in 5C).
Microwave 2-Slit Experiment

These are wavelengths used by microwave ovens, cell phones, radar, etc.
Monochromatic laser light of wavelength 0.6 \( \mu \text{m} \) is incident on two slits separated by 5 \( \mu \text{m} \), as in the figure. Is the interference at point P

A. Constructive?
B. Destructive?
C. Halfway between constructive and destructive?
If the two slits are moved closer together, then the distance between peaks 0 and 1 on the screen will

A. Decrease
B. Increase
C. Remain constant
If blue light is used instead of red, then the distance between peaks 0 and 1 on the screen will

A. Decrease
B. Increase
C. Remain constant
2-Slit Experiment with White Light

Slit spacing = 0.50 mm
Screen distance = 2.5 m

400 nm
700 nm

White

2.0 mm
3.5 mm
Laser 2-Slit Experiment

Helium-Neon Laser

Double Slits
Example

Glass of refractive index $n=1.5$ covers the lower slit of a 2-slit experiment with wavelength $\lambda=550$ nm, as suggested in the figure. If the glass has a thickness of 6.05 $\mu$m and the distance between slits is 0.02 mm, what is the angle to the first interference maximum?
Clicker Problem

In a two-slit interference experiment, if we cover one slit, how does the light intensity at the central maximum change?

A: the same
B: ½ as bright
C: ¼ as bright
D: just a little brighter
Intensity versus Amplitude

Intensity \( I \propto (\text{Amplitude})^2 = E^2 \)

When there are multiple waves

- If they are **coherent**, we add **amplitudes**
  - COHERENCE = fixed phase relationship
- If they are **incoherent**, we add **intensities**

Two coherent beams:

\[
E = E_1 + E_2 \quad \text{(add amplitudes)}
\]

\[
I \propto E^2 = (E_1 + E_2)^2 = E_1^2 + E_2^2 + 2E_1E_2
\]

Two incoherent beams:

\[
I = I_1 + I_2 \quad \text{(add intensities)}
\]

If \( E_1 = E_2 \) \( \rightarrow I_{\text{coherent}} = 4 \times I_{1,2} \)

This cross term can be positive or negative, and it averages to zero for incoherent beams
Adding Wave Amplitudes

Difference in distance between the two paths:
\[ \ell_2 - \ell_1 = d \sin \theta \]

At point P the wave functions from the two slits look like
\[ E_1(t) = E \cos(k \cdot \ell_1 - \omega t) \]
\[ E_2(t) = E \cos(k \cdot (\ell_1 + d \sin \theta) - \omega t) \]

The resultant wave is the sum of these two functions, which differ in phase by an amount
\[ \delta = k d \sin \theta = 2\pi \frac{d}{\lambda} \sin \theta \]

The two cosine functions can be added together using trig identities, or more easily by using “phasors” and vector addition.
Phasor Representation

- The red vector has a constant length equal to the amplitude of the wave, but its direction rotates with angular velocity $\omega$.
- The horizontal component of the rotating red vector is the wave function at a particular point in space (i.e. a point on the screen): $E \cos(\omega t + \delta)$

(Using phasors is equivalent to representing the amplitudes and phases by complex numbers)
Adding Two Phasors

Light wave functions at the point P of interest:

\[ E \cos(\omega t) \]
\[ E \cos(\omega t + \delta) \]

The horizontal component of the resultant vector sum of the red and blue phasors is, by the rules of vector addition, just the sum of the two horizontal components:

\[ E \cos(\omega t) + E \cos(\omega t + \delta) \]

Hence we can use vector addition to add phasors together, and the horizontal component will be the desired sum of wave functions.
Vector Addition

The exterior angle $\delta$ is equal to the sum of the two opposite interior angles: $2\phi$.

$$\phi = \frac{\delta}{2}$$

The length of the resultant vector is

$$2E \cos \phi = 2E \cos \frac{1}{2} \delta$$

and its angle with the horizontal is

$$\omega t + \phi = \omega t + \frac{1}{2} \delta$$

Hence:

$$E \cos(\omega t) + E \cos(\omega t + \delta) = 2E \cos\left(\frac{1}{2} \delta\right) \cdot \cos(\omega t + \delta/2)$$
Average Intensity

The intensity is proportional to the square of the amplitude, but we do not perceive the rapid oscillations (~$10^{14}$ Hz!). We see the constant time average:

$$\left\langle (\cos(\omega t + \delta/2))^2 \right\rangle = \frac{1}{2}$$

Hence the observed intensity is

$$I = I_0 \cos^2(\delta/2) = I_0 \cos^2\left(\frac{\pi d \sin \theta}{\lambda}\right)$$

Where $I_0$ is the intensity at $\theta=0$. 