Physics 5B
Lecture 24, March 12, 2012

Chapter 35, Single-Slit Diffraction; Instrument Resolution
Single Slit Fraunhofer Diffraction

The screen is very far away compared with the slit width.

\[ D \sin \theta = m\lambda \]
Single Slit Intensity Pattern

The amplitude on the screen at angle $\theta$ is the sum of contributions from each part of the slit:

$$E(t, \theta) = \sum_{i=1}^{N} \frac{E_0}{N} \cos(i \cdot \Delta \beta - \omega t)$$

where $\Delta \beta = \frac{2\pi}{\lambda} \Delta y \cdot \sin \theta$

The exact result follows from taking the limit $N \to \infty$.

$$\frac{1}{N} = \frac{\Delta y}{D} \to \frac{dy}{D} \quad \text{and} \quad i \cdot \Delta y \to y$$

$$E(t, \theta) = \frac{E_0}{D} \int_{0}^{D} \cos \left[ \left( \frac{2\pi}{\lambda} \sin \theta \right) \cdot y - \omega t \right] dy = E_0 \frac{\sin \beta/2}{\beta/2} \cos \left( \frac{\beta}{2} - \omega t \right)$$

where $\beta = \frac{2\pi}{\lambda} D \sin \theta$

Note: this integral is easy if it is written as the real part of a complex integral, but the textbook avoids complex integrals in favor of phasors and a geometric construction.
Single Slit Intensity Pattern

Approximating the integral as a sum of a finite number of phasors.

(a) At center, $\theta = 0$.

(b) Between center and first minimum.

(c) First minimum, $E_\theta = 0$ ($\beta = 2\pi = 360^\circ$).

(d) Near secondary maximum.
Intensity Pattern

\[ I(\theta) = I_0 \left( \frac{\sin \beta/2}{\beta/2} \right)^2 \]

where \( \beta = \frac{2\pi}{\lambda} D \sin \theta \)
Single-Slit Intensity Pattern

\[ I(\theta) = I_0 \left( \frac{\sin \beta/2}{\beta/2} \right)^2 \quad \text{where} \quad \beta = \frac{2\pi}{\lambda} D \sin \theta \]

\[ \lambda = 0.55 \, \mu\text{m} \]
\[ D = 40\lambda \]

Note that the secondary peak is not located exactly where
\[ D \cdot \sin \theta = \frac{3\lambda}{2} \]
Two-Slit Interference Pattern

\[ I(\theta) = I_0 \left( \frac{\sin \beta/2}{\beta/2} \right)^2 \left( \cos \frac{\delta}{2} \right)^2 \]

where \( \beta = \frac{2\pi}{\lambda} D \sin \theta \), \( \delta = \frac{2\pi}{\lambda} d \sin \theta \)

\[ d = 6D = 60\lambda \]
The maxima of a 2-slit interference pattern occur when
\[
\sin \theta_m^{\text{max}} = \frac{m \lambda}{d} \quad m = 0, 1, 2\ldots
\]
The minima of a single-slit diffraction pattern occur when
\[
\sin \theta_n^{\text{min}} = \frac{n \lambda}{D} \quad n = 1, 2, 3\ldots
\]
For the following interference pattern, what is the ratio of the slit spacing \(d\) to the slit width \(D\)?

\[
\frac{d}{D} = ?
\]
A. 2
B. 5
C. 6
D. 10
Problem 35-21

In a double slit experiment, \( d=5.00 \) \( D=40.0 \lambda \). Compare (as a ratio) the intensity of the third-order interference maximum with that of the zero-order maximum.
Circular Aperture

(a) Light from a single point source at \( \infty \)

(b) Light from two point sources at \( \infty \)

Rayleigh Criterion

Intensity

\[
\frac{1.22\lambda}{D}
\]

\[
0
\]

\[
\frac{1.22\lambda}{D}
\]
Intensity from a Circular Aperture

Plane waves incident from the left on a circular hole of radius $a$. The screen is very far away (Fraunhofer).

The wave passing through the square $dx\,dy$ will contribute the following to the image at angle $\theta$.

$$E(\theta) = \frac{E_0}{\pi a^2} \int_{-a}^{a} \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} \cos \left[2\pi \frac{x \cdot \sin \theta}{\lambda} - \omega t \right] dx \, dy$$

$$= \frac{E_0}{\pi^2} \frac{\lambda}{a \sin \theta} \int_{-1}^{1} \sin \left[\frac{2\pi}{\lambda} a \sin \theta \sqrt{1-u^2} \right] du$$

Do this last integral numerically with MathCad.
Integrate numerically to find the wave amplitude observed at an angle theta:

\[ E(\theta) := E_0 \frac{\lambda}{a \sin(\theta)} \int_{-1}^{1} \sin \left( \frac{2\pi a \sin(\theta)}{\lambda} \sqrt{1 - u^2} \right) du \]

MathCad does this integral numerically 200 times, once for each value of theta, in less than a second to make the plot below.

\[ \theta_{\min} = \frac{1.22\lambda}{D} \]

The intensity is zero at 1.2197
A crude camera is made from a box with film on one inside surface and a pinhole of diameter $D$ on the facing surface, a distance $L$ from the film.

You wish to take a picture of a still object far away. The picture can be made most sharp (least blurriness) by choosing the diameter $D$ how?

A. Make $D$ large.
B. Make $D$ as small as possible (increasing the exposure time to compensate for the reduced light).
C. There is an optimal value of $D$ below which the blurriness will increase.
If $L=10$ cm, what is the optimum pinhole diameter needed to obtain the sharpest image of a distant point of 500 nm light?

The angular size of the diffraction pattern from the pinhole is

$$2 \times 1.22 \frac{\lambda}{D}$$

The angular size of the direct, undiffracted image is

$$\frac{D}{L}$$

Choose $D$ to make the two comparable:

$$2 \times 1.22 \frac{\lambda}{D} = \frac{D}{L} \quad D = \sqrt{2.44 \lambda L} = 0.35 \text{ mm}$$
You want to make a spy satellite that can read a license plate on a car from an orbital altitude of 500 km. What diameter of lens or mirror would be required?

Assume that 1.0 cm resolution is needed in order to read the letters and numbers and that the wavelength is 500 nm.

Answer: 30 m. The largest space telescope mirror is <1/10 that diameter. Furthermore, the resolution would be severely limited by atmospheric turbulence, so reading a license plate would be impossible.