Damped Harmonic Oscillator

\[ T := 2 \]  
Period of the oscillator if the friction is zero

\[ \gamma := 0.35 \]  
Measure of the friction

\[ \omega := \frac{2 \cdot \pi}{T} = 3.142 \]

\[ \omega_1 := \sqrt{\omega^2 - \gamma^2} = 3.122 \]

\[ \frac{A}{\omega_1} = \frac{1}{\omega_1} \quad \varphi := -\frac{\pi}{2} \]  
Constants of integration (follow from the initial conditions)

\[ x(t) := A \cdot e^{-\gamma t} \cos(\omega_1 t + \varphi) \]

Underdamped Oscillation
The critically damped case:

\[ \gamma_2 := \omega \]

\[ B := 0 \quad C := 1 \]

These follow from the initial conditions

\[ x_c(t) := (B + C \cdot t) \cdot e^{-\gamma_2 \cdot t} \]

The overdamped case:

\[ \gamma_3 := 2 \cdot \omega \quad \omega_2 := \sqrt{\gamma^2 - \omega^2} = 5.441 \]

\[ A_2 := \frac{1}{2 \cdot \omega_2} = 0.092 \quad A_1 := -A_2 \]

\[ x_o(t) := \left( A_1 \cdot e^{-\omega_2 \cdot t} + A_2 \cdot e^{\omega_2 \cdot t} \right) \cdot e^{-\gamma_3 \cdot t} \]

Examples starting from equilibrium