Building a Saw-Tooth Wave from Sine Waves

\( \lambda := 2 \pi \)
Lambda is the wavelength of the sawtooth wave

\( n := 1, 2 \ldots 11 \)
Set of harmonics to use (both odd and even are needed)

\( k_n := n \frac{2 \pi}{\lambda} \)
Wave numbers, one for each harmonic

\( b_n := \frac{(-1)^{n+1}}{n \cdot \pi} \)
"Magic" formula for the amplitudes. (This is derived from Fourier analysis.)

Exact sawtooth wave function, for comparison:

\[
F_a(x) := \begin{cases} 
\frac{x}{\lambda} & \text{if } \frac{x}{\lambda} \geq \frac{-\lambda}{2} \wedge \frac{x}{\lambda} \leq \frac{\lambda}{2} \\
0 & \text{otherwise}
\end{cases}
\]

The individual sine waves

\[ f(n, x) := b_n \sin(k_n x) \]

Truncated Fourier series to approximate the saw-tooth wave.
Saw-Tooth Wave and Fourier Approximation

\[ F(x) \]
\[ F_a(x) \]

\[
\frac{x}{\lambda}
\]

Fourier Approximation

\[ F(x) \]

\[
\frac{x}{\lambda}
\]