Building a Triangle Wave from Cosine Waves

\[ A := 1 \] Amplitude of the triangle wave

\[ \lambda := 2 \cdot \pi \] Lambda is the wavelength of the triangle wave

\[ n := 1, 2 \ldots 7 \] Set of harmonics to use

\[ k_n := n \cdot \frac{2 \cdot \pi}{\lambda} \] Wave numbers, one for each harmonic

Exact triangle wave function, for comparison:

\[ F_a(x) := \begin{cases} 
\frac{2A}{\lambda} x & \text{if } x \geq 0 \land x \leq \frac{\lambda}{2} \\
\frac{2A}{\lambda} (\lambda - x) & \text{otherwise}
\end{cases} \]

\[ a_n := \frac{2A}{(\pi n)^2} \left[ (-1)^n - 1 \right] \] "Magic" formula for the amplitudes. (This is derived from Fourier analysis.) Note that only the odd ones are non-zero.

\[ a_0 := A \] a0 is twice the average value of the function.
\[ f(n, x) := a_n \cdot \cos\left(k_n \cdot x\right) \]  \text{The individual cosine waves}

\[ F(x) := \frac{a_0}{2} + \left(\sum_n f(n, x)\right) \]  \text{Truncated Fourier series to approximate the saw-tooth wave.}

Some Individual Cosine Waves

Triangle Wave and Fourier Approximation
Fourier Approximation

\[ F_x(\lambda) \]

The graph shows a function \( F(x) \) that approximates the behavior of \( F_x(\lambda) \) over the interval \([-1, 1]\). The function appears to be symmetric around the origin and reaches its maximum at \( \pm 0.5 \).