Final Exam
December 17, 1998

• Closed book. No notes.
• Calculators with cleared memory are okay.
• Show your work on all calculations.

You should have 5 sheets and 10 pages, with 21 problems. All numerical constants should be assumed to be accurate to two significant figures.

Equations for motion in one dimension with constant linear or angular acceleration:

\[ x = x_0 + v_0 t + \frac{1}{2} a t^2 \]
\[ x = x_0 + \frac{1}{2} (v_0 + v) \cdot t \]
\[ v = v_0 + a t \]
\[ v^2 = v_0^2 + 2a(x - x_0) \]
\[ \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha \cdot t^2 \]
\[ \theta = \theta_0 + \frac{1}{2} (\omega + \omega_0) \cdot t \]
\[ \omega = \omega_0 + \alpha \cdot t \]
\[ \omega^2 = \omega_0^2 + 2\alpha (\theta - \theta_0) \]

Newton’s second law:
\[ \vec{a} = \frac{1}{m} \cdot \vec{F}_{\text{net}} \quad \text{or} \quad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \]
\[ \vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} \quad \text{or} \quad \tau = I\alpha \]

Torque: \( \tau = r \times F \)

Moment of Inertia:
\[ I = \sum_i m_i r_i^2 \quad \text{, solid cylinder: } I = \frac{1}{2} MR^2 \quad \text{, solid sphere: } I = \frac{2}{5} MR^2 \]

Friction:
\[ f_k = \mu_k F_N \quad f_s \leq \mu_s F_N \]

Circular motion:
\[ a_{\text{rad}} = \frac{v^2}{r} = r \omega^2 \]
\[ a_{\text{tan}} = r \alpha \]
\[ v = r \omega \]
\[ s = r \theta \]

Work:
\[ dW = F \cdot ds \]
\[ F_x(x) = -\frac{dU}{dx} \]

Mechanical Energy:
\[ E = U + K \quad K = \frac{1}{2} mv^2 \quad \text{or} \quad K = \frac{1}{2} I\omega^2 \]

Power:
\[ P = \frac{dW}{dt} = F \cdot v \]

Momentum:
\[ p = mv \quad \text{Angular momentum: } L = r \times p \quad \text{or} \quad L = I\omega \]

Impulse:
\[ J = \Delta p = F_{\text{avg}} \cdot \Delta t \]

Mass on spring:
\[ F = -kx \quad U = \frac{1}{2} kx^2 \]

Gravity:
\[ g = 9.8 \text{ m/s}^2, \quad G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \]
\[ U = mgh \quad \text{or} \quad U = -G \frac{m_1 m_2}{r} \quad F = G \frac{m_1 m_2}{r^2} \]

Center of mass:
\[ r_{\text{cm}} = \frac{\sum_i m_i r_i}{\sum_i m_i} \]

Rocket propulsion:
\[ F = -v_{\text{ex}} \frac{dm}{dt} \quad v = v_0 = v_{\text{ex}} \ln \frac{m_0}{m} \]

Simple harmonic oscillator, spring:
\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad \text{pendulum: } f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \quad \omega = 2\pi f \quad \text{and} \quad T = \frac{1}{f} \]

Simple harmonic oscillator:
\[ x(t) = A \cos(\omega t + \phi) \]
(2 pnts) Suppose that you wish to tip a large packing crate so that you can put a hand truck under one edge of it. Assume that the crate doesn’t slide along the floor and that it tips about the point P. Where on the crate and in what direction should you push (or pull) in order to use the smallest force to tip the crate?

[Diagram of a crate with points X, Y, A, B, C, and E labeled]

a) Push on point X in the direction A.
b) Push on point X in the direction B.
c) Pull up on point X in the direction C.
d) Pull on point Y in the direction E.

2. (2 pnts) See the picture below. The flywheel A is rotating on the end of a long shaft that is fixed on a pivot B, which allows the shaft to pivot side to side and up and down. At the other end of the shaft is fixed a counterweight C that precisely balances the weight of the flywheel. The flywheel is rotating rapidly, with the top of the wheel moving as indicated by the white arrow (equivalently, note the direction of the angular velocity vector \( \omega \)). If you now push lightly on the counterweight with your finger in the direction indicated by the vector \( F \), in which direction will the flywheel move?

[Diagram of a flywheel with a counterweight and a vector F]

a) Up (out of the page)
b) Down (into the page)
c) In direction \( e \).
d) In direction \( f \).

3. (2 pnts) A hypothetical planet has twice the radius of the Earth and also twice the mass of the Earth. What is the magnitude of the acceleration of gravity at the surface of this planet?

a) 4.9 m/s\(^2\)
b) 9.8 m/s\(^2\)
c) 19.6 m/s\(^2\)
d) 29.4 m/s\(^2\)

4. (2 pnts) You are swinging a ball over your head in a horizontal circle at the end of a string. You quickly pull on the string to shorten the length between your hand and the ball to half of its original length. Assuming that air resistance is negligible, if the initial speed of the ball is \( v \), then its final speed is

a) \( \frac{v}{2} \)
b) \( v \)
c) \( 2v \)
d) \( 4v \)
5. (2 pnts) The space shuttle is 300 km above the surface of the earth traveling in a circular orbit when it fires its engines for a few seconds such that the thrust is applied in the direction of its instantaneous velocity.

![Image of space shuttle with thrust and Earth]

After firing the engines, the shuttle
a) takes a shorter time than before to travel once around the earth.
b) takes the same time as before to travel once around the earth.
c) takes a longer time than before to travel once around the earth.

6. (2 pnts) The bowling pin shown below has a uniform density.

![Image of bowling pin]

When suspended from a 1.0 m long string tied to point A it swings back and forth as a pendulum with period $T_A$. When the string is tied instead to point B the period is $T_B$. Which of the following must be true?

a) $T_A > T_B$
b) $T_A = T_B$
c) $T_A < T_B$

7. (2 pnts) An interplanetary probe launched by NASA is orbiting the sun in the elliptical orbit shown below. At which point in the orbit is its speed the maximum?

![Image of elliptical orbit with sun and points A, B, C, D]

a) Point A  b) Point B  c) Point C  d) Point D
e) It is the same at all points in the orbit.

8. (2 pnts) At which point in the orbit is its angular momentum maximum?

a) Point A  b) Point B  c) Point C  d) Point D
e) It is the same at all points in the orbit.
9. (2 pnts) A submerged submarine is moving at constant velocity through the water. The net force acting on it is
   a) zero 
   b) the force of water resistance 
   c) its surface area times the pressure of the water. 
   d) the thrust from the propeller minus its weight.
10. (2 pnts) Consider a tug-of-war between Team A and Team B. Team A is winning the contest by pulling Team B steadily toward them at constant velocity. Which statement is true regarding the magnitudes of the forces exerted on the rope?
   a) Team A is exerting a larger force on the rope than is Team B. 
   b) Team B is exerting a larger force on the rope than is Team A. 
   c) Team A and Team B are exerting equal forces on the rope.
11. (4 pnts) Consider the following graph of the potential energy function of a particle acted on by a conservative force (and no other force).

   ![Graph of U(x) vs x]

   a) Which of the points labeled A through G are points of unstable equilibrium?
   b) About which points could a particle execute approximately harmonic oscillations?
   c) Suppose that a particle is moving in the direction of positive $x$ with total energy greater than the value of $U(x)$ at point B. At which point A through G would the velocity of the particle be maximum?
   d) What is the direction of the force at point D? Positive $x$ or negative $x$?

**Read these instructions!** In each of the following TRUE/FALSE problems, if you choose FALSE, then you must modify the statement such that it is completely true (and still relevant to the given situation) by inserting and/or deleting a few words or a phrase.

12. (2 pnts) Suppose that a super ball and a steel ball collide head on in outer space and that before the collision the speed of the steel ball was twice that of the super ball. Then during the collision the magnitudes of the changes in momentum of the two balls are equal.
   a) TRUE 
   b) FALSE

13. (2 pnts) The magnitude of the gravitational force between any two objects that are comparable in size to the distance between them is given simply by $Gm_1m_2/r^2$, where $r$ is the distance between their centers of mass.
   a) TRUE 
   b) FALSE
14. (2 pnts) If you exert a force of magnitude $F$ on an object, the magnitude of the force that it exerts back on you depends upon the acceleration of the object relative to you.
   a) TRUE
   b) FALSE

15. (2 pnts) Suppose that a car with negligible rolling friction accelerates at constant power. In that case, the car’s speed will be increasing while its acceleration is decreasing in magnitude.
   a) TRUE
   b) FALSE

16. (6 pnts) An object is moving along the $x$ axis according to the following graph of position versus time. The continuous curve is composed of a straight line from 0 to 2, a parabola from 2 to 4, another parabola from 4 to 8, and another straight line following that. Sketch below the corresponding graphs of velocity versus time and acceleration versus time. Label the zero points on your graphs.
17. (12 pts) A block with a 4.0 kg mass is connected to each end of a rope of negligible mass, which passes over a massive pulley, with M=10 kg and radius R=0.10 m. The pulley is a uniform solid cylinder. One block slides on an inclined plane of angle 30°, while the other is suspended in air, as shown in the figure below, at an initial height of 2.0 m above the ground. The rope does not slip on the pulley, but there is no significant friction in the axle of the pulley or between the block and the plane. If the system is released from rest, what will be the speed of the blocks just before the suspended block hits the ground? Use energy methods to solve the problem.
18. (12 pnts) A 4.0 m long pole of weight 200 N is held in a vertical position, with one end resting on the ground, by a guy wire connected to the center of the pole and making an angle of 60° with the horizontal and a horizontal rope connected to the top of the pole. The wire and rope are of negligible mass. Given that the rope has a tension of 50 N, calculate the tension in the guy wire and the \( x \) and \( y \) components of the force exerted by the ground on the pole. Assume that the upward direction is the positive \( y \) direction, while the positive \( x \) direction is to the right. The signs of the components of the force are important.
19. (14 pts) Block A, of 5.0 kg mass, and Block B, of 10 kg mass, are resting on a frictionless surface. Each has a massless spring of spring constant \(k=500 \text{ N/m}\) between it and the wall, as shown in the figure below. The blocks are not connected to the springs, which are initially in equilibrium. Block A is displaced to the left by 0.25 m, compressing the spring, and then released. It slides to the right, striking Block B in a completely inelastic collision, after which the two blocks move together.

\[k=500 \text{ N/m}\]

\[\text{A} \quad 5 \text{ kg} \quad \text{B} \quad 10 \text{ kg}\]

a) What is the speed of Block A just before the collision?

b) What is the speed of the two blocks immediately after the collision, before the second spring has compressed any significant amount?

c) What is the maximum distance that Block B moves to the right?

d) How much time passes between the collision and the moment when Block B first returns to its initial position?
20. (12 pnts) An 8.0 kg block is connected by a massless rope to a 4.0 kg block. The 8.0 kg block is pushed away from the other block until the two are moving together with a speed of 3.0 m/s, at which point they are released.

\[ \text{V} \]

\[
\begin{array}{c}
\text{8 kg} \\
\text{4 kg}
\end{array}
\]

a) How far do they go before coming to rest if the coefficient of kinetic friction between block and floor is 0.20 for the 8.0 kg block and 0.35 for the 4.0 kg block?

b) How large is the tension in the rope after they are released and before they come to rest?
21. Orbital dynamics:
   a) (10 pts) Europa is one of the four large moons of Jupiter. Astronomical observations show that it is in a circular orbit about Jupiter of radius \( r = 6.7 \times 10^8 \) m with a period of \( T = 310,000 \) seconds. From these data, calculate the mass of Jupiter.

   b) (2 pts) A space probe is put into an elliptical orbit around Jupiter, as indicated in the diagram below (ignoring all possible effects of gravitational force between the moons and the space probe). Here, \( r_1 \) and \( v_1 \) are the distance from Jupiter and the speed at Point A, the point of closest approach, and \( r_2 \) and \( v_2 \) are the corresponding quantities at Point B, the furthest point in the orbit. Write down at least one of two equations between \( r_1, v_1, r_2, \) and \( v_2 \) that follow from conservation laws (your equations could also contain physical constants such as masses and \( G \)). Two points extra credit if you can get both equations correct and the extra credit doesn’t put you over 100 points total.