Physics 6A  Introduction to Physics  Winter 2001

Final Exam
March 17, 2001

• Closed book. No notes.
• Calculators with cleared memory are okay.
• Show your work on all calculations.

You should have 5 sheets and 10 pages, with 21 problems.
All numerical constants should be assumed to be accurate to two significant figures.

Equations for motion in one dimension with constant linear or angular acceleration:
\[ x = x_0 + v_0 t + \frac{1}{2} a t^2 \quad \quad x = x_0 + \frac{1}{2} (v_0 + v) \cdot t \quad \quad v = v_0 + at \quad \quad v^2 = v_0^2 + 2a \cdot (x - x_0) \]
\[ \theta = \theta_0 + \frac{1}{2} \alpha \cdot t^2 \quad \quad \theta = \theta_0 + \frac{1}{2} (\omega + \alpha) \cdot t \quad \quad \alpha = \omega_0 + \alpha \cdot t \quad \quad \alpha^2 = \omega_0^2 + 2\alpha \cdot (\theta - \theta_0) \]
Newton's second law:
\[ a = \frac{1}{m} \cdot \vec{F}_{\text{net}} \quad \text{or} \quad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \quad \quad \tau_{\text{net}} = \frac{d\vec{r}}{dt} \quad \text{or} \quad \tau = I\alpha \]
Torque: \[ \vec{\tau} = \vec{r} \times \vec{F} \]
Moment of Inertia:
\[ I = \sum_i m_i r_i^2 \]
Solid cylinder: \[ I = \frac{1}{2} MR^2 \], solid sphere: \[ I = \frac{2}{5} MR^2 \], stick of length \( l \) about one end: \[ I = \frac{1}{3} Mr^2 \]
Friction: \[ f_k = \mu_k F_N \quad f_r \leq \mu_s F_N \]
Circular motion:
\[ a_{\text{circ}} = r \omega^2 \quad \quad a_{\text{cen}} = r \alpha \quad \quad \omega = r \alpha \quad \quad s = r \theta \]
Work:
\[ dW = \vec{F} \cdot d\vec{s} \]
\[ F_x(x) = -\frac{dU}{dx} \]
Mechanical Energy:
\[ E = U + K \quad \quad K = \frac{1}{2} mv^2 \text{ or } K = \frac{1}{2} \omega^2 \quad \quad \text{Power: } P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} \]
Momentum:
\[ \vec{p} = m\vec{v} \quad \quad \text{Angular momentum: } \vec{L} = \vec{r} \times \vec{p} \text{ or } \vec{L} = I\vec{\omega} \quad \quad \text{Impulse: } \vec{J} = \Delta \vec{p} = \vec{p}_{\text{end}} - \vec{p}_{\text{start}} \]
Mass on spring:
\[ F = -kx \quad \quad U = \frac{1}{2} kx^2 \]
Gravity:
\[ g = 9.8 \text{ m/s}^2 \quad \quad \text{or } g = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^2 \]
\[ U = mgh \quad \text{ or } U = -G \frac{m_1 m_2}{r} \quad \quad F = G \frac{m_1 m_2}{r^2} \]
Simple harmonic oscillator, spring:
\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad \quad \text{pendulum: } f = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \quad \quad \omega = 2\pi f \text{ and } T = \frac{1}{f} \]
Simple harmonic oscillator:
\[ x(t) = A \cos(\omega t + \phi) \]
\[ \frac{d^2 x}{dt^2} + \omega^2 x = 0 \]
1. (2 pts) See the picture below. The flywheel A is rotating on the end of a long shaft that is fixed on a pivot B, which allows the shaft to pivot side to side and up and down. At the other end of the shaft is fixed a counterweight C that precisely balances the weight of the flywheel. The flywheel is rotating rapidly, with the top of the wheel moving as indicated by the white arrow (equivalently, note the direction of the angular velocity vector \( \vec{\omega} \)). If you now push lightly on the counterweight with your finger in the direction indicated by the vector \( \vec{F} \), in which direction will the flywheel move?

\[ \text{torque is out of the page, hence so is } \vec{F} \]

a) Up (out of the page)
b) Down (into the page)
c) In direction \( \vec{\omega} \)
d) In direction \( \vec{F} \)

2. (2 pts) A hypothetical planet has half the radius of the Earth and half the mass of the Earth. What is the magnitude of the acceleration of gravity at the surface of this planet?
   a) 4.9 m/s²
   b) 9.8 m/s²
   c) 9.6 m/s²
   d) 29.4 m/s²

3. (2 pts) The space shuttle is 400 km above the surface of the earth traveling in a circular orbit when it fires its engines for a few seconds such that the thrust is applied opposite the direction of its instantaneous velocity.

After firing the engines, the shuttle
a) takes a shorter time than before to travel once around the earth.
b) takes the same time as before to travel once around the earth.
c) takes a longer time than before to travel once around the earth.
4. (2 pts) An interplanetary probe launched by NASA is orbiting the sun in the elliptical orbit shown below. At which point in the orbit is its kinetic energy the minimum?

![Elliptical orbit diagram]

a) Point A  b) Point B  c) Point C  d) Point D

e) It is the same at all points in the orbit.

5. (2 pts) If the size of the orbit were doubled (keeping the same shape), by what factor would the period of the orbit change?

a) \(\frac{1}{2}\)  b) 2  c) \(2\sqrt{2}\)  d) 4
e) Impossible to tell without knowing the mass of the probe.

6. (2 pts) What happens to the frequency of oscillation of a mass on a spring if both the spring constant and the mass are increased by the same factor?

a) It increases.  b) It decreases.  c) It remains unchanged.
d) It is impossible to tell without knowing the original mass and spring constant.

7. (2 pts) Two projectiles are propelled with the same initial vertical speed but with different initial horizontal speeds. If air resistance is negligible, how will their time in the air and horizontal flight distance compare?

a) They will remain in the air different amounts of time but will travel the same distance.  b) They will remain in the air different amounts of time and travel different distances.  c) They will remain in the air the same amount of time and travel the same distance.
d) They will remain in the air the same amount of time but travel different distances.

8. (2 pts) An object is weighed on Earth using a balance and found to have a mass of 3.0 kg. What is its mass on the moon, where the acceleration of gravity is 1/6 the value on Earth? Weights not mass, change.

a) 0  b) 0.5 kg  c) 3.0 kg  d) 18 kg

9. (2 pts) Two identical springs are compressed 15 cm by two different people (a and b) at different varying rates. These two graphs show the distance that the spring is compressed versus time for each person.

![Graphs of compression versus time]

a) Person a does more work than person b.  b) Person b does more work than person a.
c) Both people do the same amount of work.  d) The force is not constant with time, so we cannot calculate the amount of work done in either case.

10. (2 pts) The magnitude of the acceleration of a harmonic oscillator is maximum

a) when the oscillator is at its equilibrium position.  b) when the speed is maximum.
c) when the speed is zero.  d) always, since the acceleration of a harmonic oscillator is constant.

At maximum amplitude, the force is maximum.
11. (2 pts) Two wheels initially at rest roll the same distance without slipping down identical inclined planes starting from rest. Wheel B has twice the radius but the same mass as wheel A. All the mass is concentrated in their rims, so that the rotational inertias are I=MR². Which has more translational kinetic energy when it gets to the bottom?
   a) Wheel A.
   b) Wheel B.
   c) The translational kinetic energies are the same.
   d) Not enough information is given to answer the question.

12. (2 pts) A person swings on a playground swing. When the person sits, the swing plus person oscillate back and forth at the natural frequency. If, instead, the person stands on the swing, the natural frequency of the swing
   a) is increased.
   b) is reduced.
   c) remains unchanged.

13. (2 pts) A uniform solid sphere, a uniform solid cylinder, and a hollow cylinder are simultaneously released at the top of an inclined plane. They all roll without slipping and with negligible rolling friction.
   a) Which will be the first to arrive at the bottom?  (i) sphere (ii) solid cylinder (iii) hollow cylinder.
   b) Which will be the last to arrive at the bottom?  (i) sphere (ii) solid cylinder (iii) hollow cylinder.

14. (2 pts) The net torque on an object always points in the direction of
   a) The angular acceleration of the object.
   b) The time derivative of the angular momentum.
   c) The net applied force.
   d) The moment of inertia.

15. (2 pts) The ocean tides of the Earth are due to
   a) The fact that water is less dense than rock and therefore is less attracted to the Sun.
   b) The fact that the Sun is always pulling on the Earth in a different direction from the pull of the Moon.
   c) The variation in the gravitational forces from the Sun and Moon from one side of the Earth to the other.
   d) The fact that water will always tend to gravitate toward the side of the Earth closest to the Sun and Moon.
16. One end of a uniform stick of length \( L \) is placed upon a point such that it is leaning at an angle \( \theta \) from the vertical, as indicated in the figure. It is then released and allowed to topple over.

a) (4 pts) Derive an expression for the torque exerted by gravity on the bar, expressed in terms of \( m \), \( g \), \( \theta \), and \( L \). Take the axis of rotation to be at the lower end of the bar.

\[
\tau = F_g \cdot L/2 = mg \sin \theta \cdot \frac{L}{2}
\]

\[
\tau = \frac{1}{2} mg L \sin \theta
\]

b) (4 pts) Derive an expression for the angular acceleration \( \alpha \) of the stick about the point at the moment it is released, in terms of \( g \), \( \theta \), and \( L \).

2nd law: \( \tau = I \alpha \)

\[
I = \frac{1}{3} mL^2
\]

\[
\frac{1}{2} mg L \sin \theta = \frac{1}{3} mL^2 \alpha
\]

\[
\alpha = \frac{3g \sin \theta}{2L}
\]

c) (7 pts) If \( L = 2.0 \text{ m} \), what is the angular velocity \( \omega \) of the rod (rotating about the point) when it is horizontal (\( \theta = 90^\circ \)), assuming that it started out very close to vertical (\( \theta = 0^\circ \)) and with zero initial angular velocity. Assume that it does not slip off of the point, which acts as a frictionless pivot. (Hint: Do not try to do this by using formulas for motion with constant angular acceleration, as the acceleration derived in part (a) varies with the angle. Making use of a conservation law would be more appropriate.)

**Initial** \( E = mg L/2 \)

**Final** \( E = \frac{1}{2} I \omega^2 \)

\[
mg \frac{L}{2} = \frac{1}{2} I \omega^2 = \frac{1}{2} \left( \frac{1}{3} mL^2 \right) \omega
\]

\[
\omega = \sqrt{\frac{3g}{L}} = \sqrt{\frac{3 \cdot 9.8 \text{ m/s}^2}{2.0 \text{ m}}}
\]

\[
\omega = 3.8 \text{ rad/s}
\]
17. Consider a uniform horizontal bar of weight \( W = 500 \text{ N} \) supported by a cable and a wall as illustrated below. The cable attaches to the center of the bar, at a 60° angle. A 1000 N weight is suspended from the end of the bar. The opposite end rests against the wall and relies upon static friction to keep it in place.

a) (12 pts) Find the tension in the cable, the normal force of the wall on the bar, and the magnitude and direction of the friction force.

\[ \Sigma F_x = F_N - T \cos 60 = 0 \]
\[ \Sigma F_y = T \sin 60 - f - 1500 = 0 \]
\[ \Sigma T = 1000 \cdot 2 + 500 \cdot \frac{1}{2} - T \sin 60 \cdot \frac{1}{2} = 0 \]

\[ T = \frac{2500 \text{ N}}{\sin 60} = 2890 \text{ N} = 2.9 \times 10^3 \text{ N} \]

\[ F_N = T \cos 60 = 2890 \cdot 0.5 \cdot 60 = 1450 \text{ N} = 15.1 \times 10^2 \text{ N} \]

\[ f = T \sin 60 - 1500 = 2500 - 1500 = 1000 \text{ N} \]

f acts downward, as drawn

b) (3 pts) What is the minimum coefficient of static friction between bar and wall necessary to keep the bar from moving?

\[ \mu_s \geq \frac{f}{F_N} = \frac{1000 \text{ N}}{1450 \text{ N}} = 0.69 \]
18. A cable of negligible mass is wrapped around a uniform solid cylinder of mass \( m \) and radius \( R \) with a fixed axis of rotation. A block of the same mass \( m \) is tied to the free end of the cable. It is then released from a height \( h \) above the floor and falls under gravity. Assume that the cable does not slip on the cylinder as the mass falls and the cylinder turns. For the following questions, give the results in terms of \( m, g, h, \) and \( R \).

a) (7 pts) What is the tension in the cable?

\[
\begin{align*}
F &= ma \\
\Rightarrow mg - T &= ma
\end{align*}
\]
\[
\gamma = I \alpha \Rightarrow TR &= \left( \frac{1}{2} m R^2 \right) \alpha
\]
But \( \alpha = \frac{g}{R} \) if cable doesn't slip
\[
TR = \frac{1}{2} m R^2 \frac{g}{R}
\]
\[
T = \frac{1}{2} ma \Rightarrow a = \frac{2T}{m}
\]
\[
mg - T = 2T
\]
\[
mg = 3T
\]
\[
T = \frac{1}{3} mg
\]

b) (7 pts) What is the speed of the falling mass just before it strikes the floor? (Hint: you can do this either by conservation of energy or by calculating the acceleration.)

\[
\text{Initial } E_i = mgh
\]
\[
\text{Final } E_f = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 \Rightarrow \omega = \frac{g}{R}
\]
\[
E_f = \frac{1}{2} mv^2 + \frac{1}{2} \frac{1}{2} m R^2 \omega^2 \Rightarrow I = \frac{1}{2} m R^2
\]
\[
E_f = \frac{3}{4} mv^2
\]
\[
\frac{3}{4} mv^2 = mgh \Rightarrow v = \sqrt{\frac{4}{3} gh}
\]
\[
\frac{2T}{m} = \frac{2}{3} g ~ \text{and} ~ h = \frac{1}{2a} (v^2 - v_0^2) \quad v_0 = 0
\]
\[
\frac{2a}{m} = \frac{2}{3} g \quad \text{and} \quad h = \frac{1}{2a} \left( \frac{4}{3} gh \right) \quad v = \sqrt{\frac{4}{3} gh}
\]
10. Two blocks are stuck together and sliding across an ice
   rink when a firecracker explodes between them and blows
   them apart from each other. The 3.0 kg block goes in the
   x direction after the explosion with a speed of 4.0 m/s,
   while the 5.0 kg block goes in the y direction with a speed
   of 2.0 m/s.
   a) (8 pts) What was the velocity (speed and direction)
      of the two blocks before the explosion?

      Momentum is conserved
      
      After:  \( p_x = 3.0 \text{ kg} \cdot 4.0 \text{ m/s} \)
      \( p_y = 5.0 \text{ kg} \cdot 2.0 \text{ m/s} \)

      So before the explosion:  \( p_x = 12 \text{ kg} \cdot \text{m/s} \)
      \( p_y = 10 \text{ kg} \cdot \text{m/s} \)

      \( p = \sqrt{p_x^2 + p_y^2} = 15.6 \text{ kg} \cdot \text{m/s} \)

      \( v = \frac{p}{m_{\text{tot}}} = \frac{15.6 \text{ kg} \cdot \text{m/s}}{(3.0 + 5.0) \text{ kg}} = 2.0 \text{ m/s} \)

      \( \theta = \tan^{-1} \frac{p_y}{p_x} = \tan^{-1} \frac{10}{12} = 39.8^\circ \Rightarrow 40^\circ \)

   b) (6 pts) How much mechanical energy was gained by the system from the explosion?

      \( E_f = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} (5.0) (2.0)^2 + \frac{1}{2} (3.0) (4.0)^2 \)

      \( E_f = 34 \text{ J} \)

      \( E_i = \frac{1}{2} m_{\text{tot}} v^2 = \frac{1}{2} (3.0 + 5.0) (2.0)^2 = 16 \text{ J} \)

      \( \Delta E = E_f - E_i = 34 - 16 = 18 \text{ J} \)
20. (Work either Problem 20 or Problem 21.) A hypothetical planet is a uniform sphere of mass $M$ and radius $R$, except that it has a narrow, straight tunnel bored from a point on the surface all the way through the center and out to the opposite side of the sphere. A rock of mass $m$ is dropped into the tunnel from the surface at time $t = 0$. It can be shown from Newton's law of gravity (Example 12-9 in the textbook) that the gravitational force on the rock while it falls through the tunnel is directed toward the center of the planet and has a magnitude proportional to the distance from the center:

$$F = \left( \frac{GMm}{R^3} \right) r$$

a) (6 pts) Assuming that the tunnel has no air inside, then the rock will travel back and forth from one side of the planet to another in a harmonic oscillation, with $r(t) = R \cos(\omega t)$. Explain clearly why this is so, based on Newton's laws and the above information.

From 2nd law \( \Rightarrow -G \frac{Mm}{R^2} \frac{d^2r}{dt^2} = ma = m \frac{d^2c}{dt^2} \)

\[ \frac{d^2c}{dt^2} + \left( \frac{G M}{R^2} \right) c = 0 = \text{simple harmonic oscillator} \]

with \( \omega^2 = \frac{G M}{R^2} \)

b) (6 pts) Assuming that the planet has the mass and size of Earth \((M=6.0 \times 10^{24} \text{ kg} \text{ and } R=6.4 \times 10^6 \text{ m})\), how long, in seconds, will it take for the rock to go from one side of the planet to the other and return?

This is one period of oscillation.

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi \sqrt{\frac{R^3}{GM}}$$

$$= 2\pi \sqrt{\frac{(6.4 \times 10^6)^3}{(6.7 \times 10^{10} \text{ kg})(6.0 \times 10^{24} \text{ m})}} \left( \frac{m^2}{N \cdot m^2 / \text{kg}} \right)$$

$$T = 5.1 \times 10^3 \text{ s}$$
21. (Work either Problem 20 or Problem 21.) (12 pts) A super powerful spring gun is designed to 
launch a 2.0 kg ball into outer space from the surface of the moon. Given that the mass of the moon is 
$7.4 \times 10^{22}$ kg and its radius is $1.7 \times 10^6$ m, and assuming that the spring constant is \( k = 800,000 \) N/m, how 
far must the spring be compressed in order for the ball to escape from the moon (that is, to be launched 
to infinite height)?

\[
E_i = \frac{1}{2} k x_c^2 - G \frac{M m}{R} \quad \text{at surface}
\]

\[
E_f = - G \frac{M m}{R+h} \quad \text{with } h = \infty
\]

Energy is conserved

\[
E_i = E_f = 0
\]

\[
\frac{1}{2} k x_c^2 - G \frac{M m}{R} = 0
\]

\[
x_c = \sqrt{\frac{2 G M m}{k R}}
\]

\[
x_c = \sqrt{\frac{2 \cdot 6.67 \cdot 10^{-11} \text{ N m}^2/\text{kg}^2 \cdot 7.4 \cdot 10^{22} \text{ kg} \cdot 2.0 \text{ kg}}{8 \cdot 10^5 \text{ N/m} \cdot 1.7 \cdot 10^6 \text{ m}}}
\]

\[
x_c = 3.8 \text{ m}
\]