Physics 6A  Introduction to Physics  Autumn 1998  

Midterm Exam 2  November 18, 1998

- Closed book. No notes.
- Calculators with cleared memory are okay.
- Show your work on all calculations.

You should have 3 sheets and 6 pages, with 16 problems.
All numerical constants should be assumed to be accurate to two significant figures.

Equations for motion in one dimension with constant linear or angular acceleration:
\[ x = x_0 + v_0 t + \frac{1}{2} a t^2 \quad x = x_0 + \frac{1}{2} (v_0 + v) t \quad v = v_0 + at \quad v^2 = v_0^2 + 2a(x - x_0) \]
\[ \theta = \theta_0 + \alpha_0 t + \frac{1}{2} \alpha t^2 \quad \theta = \theta_0 + \frac{1}{2} (\alpha_0 + \alpha) t \quad \alpha = \alpha_0 + \alpha t \quad \alpha^2 = \alpha_0^2 + 2\alpha (\theta - \theta_0) \]

Newton's second law: \[ \vec{a} = \frac{1}{m} \vec{F}_{\text{net}} \]
Friction: \[ f_s = \mu_s F_N \quad f_s \leq \mu_s F_N \]
Circular motion: \[ a_{\text{rad}} = \frac{v^2}{r} = r\omega^2 \quad a_{\text{tan}} = r\alpha \quad v = r\omega \quad s = r\theta \]

Work: \[ dW = \vec{F} \cdot d\vec{s} \quad F_s(x) = -\frac{dU}{dx} \]
Mechanical Energy: \[ E = U + K \quad K = \frac{1}{2} mv^2 \quad \text{Power: } P = \frac{dW}{dt} = \vec{F} \cdot \vec{v} \]
Momentum: \[ \vec{p} = m\vec{v} \quad \text{Impulse: } \vec{J} = \Delta \vec{p} = \vec{F}_{\text{avg}} \cdot \Delta t \]
Mass on spring: \[ F = -kx \quad U = \frac{1}{2} kx^2 \]
Gravity: \[ g = 9.8 \text{ m/s}^2 \quad U = mgh \quad \text{Weight: } \vec{W} = m\vec{g} \]
Center of mass: \[ \vec{r}_{\text{cm}} = \frac{\sum m_i \vec{r}_i}{\sum m_i} \]
Rocket propulsion: \[ F = -v_{\text{ex}} \frac{dm}{dt} \quad v - v_0 = v_{\text{ex}} \ln \frac{m_0}{m} \]

1) (3 pts) A child's spring gun is used to launch a ping-pong ball across the room. While the spring is decompressing and still in contact with the ball,
   a) the acceleration of the ball is constant and its velocity is increasing.
   b) the acceleration of the ball is increasing and its velocity is increasing.
   c) the acceleration of the ball is decreasing and its velocity is increasing.
   d) the acceleration of the ball is constant and its velocity is constant.

2) (3 pts) A child pulls horizontally on a box placed on level ground with a force of 20 N, but the box does not move. If the box weighs 40 N, then which statement is true about the coefficient of static friction \( \mu_s \) between the box and the ground?
   a) \( \mu_s = 0.5 \)  \( \quad \mu_s = 0.5 \)
   b) \( \mu_s \geq 0.5 \)  \( \quad \mu_s \leq 0.5 \)
   c) \( \mu_s = 2.0 \)  \( \quad \mu_s = 0.5 \)
   d) \( \mu_s = 2.0 \)  \( \quad \mu_s = 2.0 \)

\[ F \leq \mu_s F_N \quad \text{so } \mu_s \geq \frac{F}{F_N} \]
3) (3 pts) Two identical boxes each are pulled at constant speed a distance of 3 m up a 35° friction-free inclined plane. Box A is pulled by a rope oriented parallel to the plane, while Box B is pulled by a rope oriented vertically upwards.

Considering the mechanical work done on the box by the person pulling the rope,

a) more work is done on Box A than on Box B.
b) more work is done on Box B than on Box A.
c) the same amount of work is done on Box A and Box B.

4) (3 pts) Two balls, A and B, with masses \( m_A = 2m_B \), are launched from identical spring guns at an upward angle of 45° from level ground. Which ball will hit the ground with a higher speed (assuming that air resistance is not an important factor)?

a) Ball A (the heavy one).
b) Ball B (the light one).
c) They hit with the same speed.

5) (3 pts) Consider two carts, of masses \( m \) and \( 2m \), at rest on an air track. If you push first one cart for 3 s and then the other for the same length of time, exerting equal force on each, the kinetic energy of the light cart is

a) larger than
b) equal to
c) smaller than

the kinetic energy of the heavy cart.

6) (3 pts) A cart moving with speed \( v \) collides with an identical stationary cart on an air track, and the two stick together after the collision. What is their speed after colliding?

a) \( v \)
b) \( 0.25v \)
c) \( 0.5v \)
d) zero
e) \( 2v \)

7) (3 pts) A golf ball is fired at a bowling ball initially at rest and bounces back elastically. Compared with the bowling ball, the golf ball after the collision has

a) larger magnitude of momentum but less kinetic energy.
b) larger magnitude of momentum and more kinetic energy.
c) smaller magnitude of momentum and less kinetic energy.
d) smaller magnitude of momentum but more kinetic energy.

8) (3 pts) Consider the situation shown at left below.

A puck of mass \( m \), moving at speed \( v \) on a frictionless air table, hits an identical puck that is fastened to a pole using a string of length \( r \). After the collision, the puck attached to the string revolves around the pole with constant angular speed. Suppose we now lengthen the string by a factor of 2, as shown on the right, and repeat the experiment. Compared to the angular speed in the first situation, the new angular speed is

a) twice as high.
b) the same.
c) half as much.
d) none of the above.
9) (3 pts) The spring in configuration (a) is stretched 0.10 m from its equilibrium length. How much will the same spring be stretched in configuration (b)?

![Diagram of two configurations with a spring and mass](image)

(a) (b)

- a) 0.05 m
- b) 0.10 m
- c) 0.20 m
- d) 0.40 m

The tension on the spring is the same in both cases.

10) (3 pts) A ball is placed on a spring that is compressed 10 cm from its equilibrium length. When released, the spring launches the ball vertically upward to a height of 3 m. At what point is the magnitude of the acceleration of the ball the greatest?

- a) Immediately after the spring is released.
- b) Just before the ball loses contact with the spring.
- c) Just after the ball loses contact with the spring.
- d) At the highest point of the trajectory.

11) (3 pts) The diagram below depicts two pucks on a frictionless table. Puck B is four times as massive as puck A. Starting from rest, the pucks are pushed across the table by two equal forces.

![Diagram of two pucks](image)

Which puck has the greater kinetic energy upon reaching the finish line?

- a) Puck A.
- b) Puck B.
- c) They both have the same amount of kinetic energy.
- d) Too little information is given to answer.

12) (3 pts) Which of the two pucks reaches the finish line first?

- a) Puck A.
- b) Puck B.
- c) They both reach the finish line at the same time.
- d) Too little information is given to answer.

A has larger acceleration than B, \( a = \frac{F}{m} \)

13) (3 pts) Which of the two pucks has the greater momentum upon reaching the finish line?

- a) Puck A.
- b) Puck B.
- c) They both have the same momentum.
- d) Too little information is given to answer.

B will have the force acting for a longer time \( \Rightarrow \) larger impulse.
14) (20 pts) A child pulls a box of mass $m$ across a horizontal floor with constant acceleration $a$ by a rope that makes an upward angle of $\theta$ with respect to the floor, as indicated in the figure below. The coefficient of kinetic friction between the floor and box is $\mu_k$.

![Diagram of a box being pulled with forces](image)

a) Draw a free body diagram indicating all of the forces acting on the block. Label the forces and angles clearly (you do not need to show the individual components of each force).

![Free Body Diagram](image)

b) Write down Newton’s second law for this situation (2 equations, one for each of the $x$ and $y$ coordinates), and solve for the rope tension $T$ in terms of only $\theta$, $g$, $\mu_k$, $a$, $m$.

\[
\begin{align*}
\text{x: } T \cos \theta - f &= ma \\
\text{y: } T \sin \theta + F_N - mg &= 0 \\
F_N &= mg - T \sin \theta \\
f &= \mu_k F_N = \mu_k mg - T \mu_k \sin \theta \\
x: T \cos \theta - \mu_k mg + T \mu_k \sin \theta &= ma \\
T \left( \cos \theta + \mu_k \sin \theta \right) &= m(a + \mu_k g) \\
T &= \frac{m(a + \mu_k g)}{\cos \theta + \mu_k \sin \theta}
\end{align*}
\]
15) (21 pts) A massless spring with $k = 2000 \text{ N/m}$ is used to launch a block of mass $m = 0.50 \text{ kg}$ sliding on a frictionless track. The track forms a hill of height $h = 0.45 \text{ m}$, as indicated in the figure below.

If the spring is compressed a distance $x_c = 0.050 \text{ m}$ and released, then
a) what is the speed $v$ of the block at the crest of the hill?

\[ E_x = \frac{1}{2} k x_c^2 \]
\[ F_f = mgh + \frac{1}{2} mv^2 \]
\[ E_x = E_f \Rightarrow \frac{1}{2} k x_c^2 = mgh + \frac{1}{2} mv^2 \]
\[ v = \sqrt{\frac{k}{m} x_c^2 - 2gh} \]
\[ = \sqrt{\frac{2000}{0.5} (0.05)^2 - 2 \cdot 9.8 \cdot 0.45} = 1.09 \text{ m/s} \]

b) What is the normal force $F_N$ exerted on the block by the track at the crest of the hill if the radius of the curvature of the hill is $r = 0.20 \text{ m}$ at that point?

\[ mg - F_N = m \frac{v^2}{r} \]
\[ F_N = m \left( g - \frac{v^2}{r} \right) \]
\[ = 0.50 \left( 9.8 - \frac{1.09^2}{0.20} \right) \]
\[ F_N = 1.93 \text{ N} \]
16) (20 pts) A hockey puck of weight 4.0 N is held by a rotating arm of length \( r = 0.50 \text{ m} \) that accelerates it from rest, starting at point A, and releases it at point B after a 90° rotation. The angular acceleration of the arm is \( \alpha = 16 \text{ rad/s}^2 \), and the coefficient of kinetic friction between the table top and the puck is \( \mu_k = 0.20 \).

![Diagram of the puck and arm](image)

(a) What is the speed of the hockey puck just after it is released by the arm?

\[
\omega^2 = \omega_0^2 + 2\alpha (\theta - \theta_0) \\
\omega = \sqrt{2 \times 16 \times \frac{\pi}{2}} \\
\omega = 7.09 \text{ rad/s}
\]

\[
v = \omega r = 7.09 \times 0.50 = 3.54 \text{ m/s}
\]

(b) How much work is done on the puck by friction as it slides across the floor and comes to rest?

\[
W = \Delta K = -\frac{1}{2}mv^2 = -\frac{1}{2} \left(\frac{4.1^2}{2.8}\right) 3.54^2 = -2.56 \text{ J}
\]

(c) How does the puck travel from the point at which it is released by the arm?

\[
d = \frac{W}{f} = \frac{W}{\mu_k mg} = \frac{W}{\mu_k m g}
\]

\[
d = \frac{2.56}{0.20 \times 4.0} = 3.20 \text{ m}
\]