Print your name: Solutions

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Signature:

Closed book; no notes. A straightedge and pencil are needed for the ray diagram. You may assume that any numerical constant provided in this exam is accurate to two significant figures. Check that your exam includes 32 problems on 10 pages. Print your name and sign your exam.

The following constants and equations may or may not be needed:
- Acceleration of gravity: $g = 9.8 \text{ m/s}^2$
- Speed of sound in air: 350 m/s.
- Speed of light in vacuum: $3.0 \times 10^8 \text{ m/s}$.
- Gas constant: $R = 8.3 \text{ J/mol-K}$
- Stefan-Boltzmann constant: $\sigma = 5.7 \times 10^{-8} \text{ W/(m}^2 \text{ K}^4 \text{)}$
- Ratio of specific heat at constant pressure to that at constant volume, for air: $\gamma = 1.4$
- Young’s modulus of steel: $Y = 20 \times 10^{10} \text{ Pa}$
- Density of water: 1000 kg/m$^3$. Specific heat of water: 4200 J/kg°C. Specific heat of ice: 2100 J/kg°C
- Heat of fusion of water: 330,000 J/kg
- Average power transmitted by a harmonic wave of amplitude $A$ and angular frequency $\omega$, on a string of tension $F$: $P = \frac{1}{2} \mu F \omega^2 A^2$
- Intensity of a sound wave: $I = \frac{1}{2} \rho B \omega^2 A^2 = \frac{P_{\text{max}}^2}{2 \rho v} = \frac{P_{\text{max}}^2}{2 \sqrt{\rho B}}$
- Relation between pressure and displacement amplitudes in a longitudinal sound wave: $P_{\text{max}} = B k A$.
- Wave equation: $\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} = 0$
- Doppler shift formula: $f_L = \frac{1 + v_L}{1 + v_s/v} f_S$.
- Angle of a shock wave: $\sin \alpha = v/v_s$
- Malus’ law: $I = I_{\text{max}} \cos^2 \phi$
- Equation for images formed by a single refracting surface: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$
- Lateral magnification of a single refracting surface: \( m = -\frac{n_a s'}{n_b s} \)

- Thin lens equation: \( \frac{1}{s} + \frac{1}{s'} = (n-1) \left[ \frac{1}{f_1} - \frac{1}{f_2} \right] \)

- Angular magnification of a magnifying glass: \( M = \frac{25 \text{ cm}}{f} \)

- Angular magnification of a telescope: \( M = -\frac{f_1}{f_2} \)

- Magnification of a compound microscope: \( M \propto \frac{1}{f_1 f_2} \)

- Intensity for a single-slit diffraction/interference pattern: \( I = I_0 \left( \frac{\sin\left(\frac{\pi a (\sin \theta)}{\lambda}\right)}{\frac{\pi a (\sin \theta)}{\lambda}} \right)^2 \)

- Intensity for a 2-slit interference pattern: \( I = I_0 \cos^2 \left( \frac{\phi}{2} \right) \left( \frac{\sin\left(\frac{\beta}{2}\right)}{\frac{\beta}{2}} \right)^2 \)

where \( \phi = \frac{2n a}{\lambda} \sin \theta \) and \( \beta = \frac{2n a}{\lambda} \sin \theta \)

- Bragg condition for constructive interference of x-rays reflected from a crystal: \( 2d \sin \theta = n \lambda \).

- Angular radius of the first dark ring of a diffraction/interference pattern from a circular aperture: \( \sin \theta_1 = 1.22 \frac{\lambda}{D} \).

- Bernoulli’s equation: \( p_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2 \).

- Poiseuille’s equation: \( \frac{dV}{dt} = \frac{\pi}{8} \left( \frac{R^4}{\eta} \right) (p_1 - p_2) \)

- Stoke’s law: \( F = 6\pi \eta rv \)

1. (1 pt) A toy boat of volume 0.020 m³ weighing 15 kg is placed on the water in a bathtub filled to the brim. What is the mass of the water that flows over the edge of the bathtub?
   a) 5 kg
   b) 10 kg.
   c) 15 kg.
   d) Not enough information is given to answer this question.

2. (1 pt) A highly viscous liquid is being pumped through a long cylindrical pipe. If the pressure difference between the two ends of the pipe is cut in half but at the same time the radius of the pipe is doubled, then the volume flow rate will increase by a factor of
   a) 2
   b) 4
   c) 8
   d) 16

3. (1 pt) The flow of a fluid through a pipe is laminar. Which one of the following actions might result in the onset of turbulence, assuming all other properties are kept constant?
   a) Use a more viscous fluid.
   b) Increase the velocity of the fluid flow.
   c) Decrease the diameter of the pipe.
   d) Use a less dense fluid.

4. (1 pt) In the case of a farsighted person (hyperopic) the image of a close (e.g. 25 cm) object
   a) focuses in a different plane for horizontal versus vertical lines.
   b) forms between the lens and the retina.
   c) would form behind the retina, on the opposite side from the lens.

5. (2 pts) A perfectly exposed photograph is taken with a 1/500 s long exposure using a 50 mm focal length lens and an f-number of 2.0. The lens is then replaced by a 135 mm focal length lens with an f-number of 2.8. How long should the exposure be in order to photograph correctly the same subject with the new lens?
   a) 1/125 s.
   b) \(1/250\) s.
   c) 1/500 s.
   d) 1/1000 s.
6. (1 pt) To obtain corrected vision, a nearsighted person (myopic) should wear glasses with
   a) converging lenses.
   b) diverging lenses.

7. (1 pt) A blood platelet drifts along with the flow of blood through an artery that is partially blocked by
deposits. As the platelet moves from the narrow region to the wider region, it experiences
   a) an increase in pressure.
   b) a decrease in pressure.
   c) no change in pressure.

8. (1 pt) Which of the following has the greatest effect on the flow rate of fluid through a pipe? That is,
   if you made a 10% change in each of the quantities below, which would cause the greatest change in
   the flow rate?
   a) The fluid viscosity.
   b) The radius of the pipe.
   c) The pressure difference between the ends of the pipe.
   d) The length of the pipe.

9. (1 pt) If you double the temperature of an object that is much hotter than its surroundings, it will
   radiate heat energy
   a) 2 times faster.
   b) 4 times faster.
   c) 8 times faster.
   d) 16 times faster.

10. (1 pt) If you reduce the thickness of a wall built from a homogenous material to one half its original
    thickness, the rate of heat loss for a given temperature difference across the thickness will
    a) become one half of its original value.
    b) remain constant.
    c) double.
    d) quadruple.

11. (1 pt) Suppose that you are trying to see a faint galaxy with your backyard refracting telescope on a
    dark, clear night. Unfortunately it isn't quite visible. To see the galaxy you need
    a) an objective lens with a larger diameter.
    b) a larger distance between the objective lens and the eyepiece.
    c) an eyepiece with a shorter focal length.
    d) an objective lens with a longer focal length.

12. (1 pt) To increase the angular magnification of your refracting telescope, you need to
    a) replace the eyepiece by one with a longer focal length.
    b) replace the eyepiece by one with a shorter focal length.
    c) keep the same objective lens and eyepiece but move them further apart.
    d) keep the same objective lens and eyepiece but move them close together.

13. (1 pt) To increase the field of view of your camera, you can
    a) replace the lens by one with the same focal length but a larger diameter.
    b) replace the lens by one with a shorter focal length
    c) replace the lens by one with a longer focal length
    d) keep the same lens and eyepiece but move it closer to the film.
14. (1 pt) To increase the depth of focus of your camera, you can
   a) Increase the diameter of the aperture (lower f-stop setting).
   b) Reduce the diameter of the aperture (higher f-stop setting).
   c) Reduce the length in time of the exposure.
   d) Move the lens closer to the film.

15. (1 pt) The picture at the right shows waves from two sources interfering. The rings are
   wave crests, and the rays show lines of constructive interference. As the two sources
   are moved closer together,
   a) the lines of constructive interference will
      move further apart.
   b) the lines of constructive interference will
      move closer together.
   c) the lines of constructive interference shown will not change, but new lines will
      appear between them.
   d) the lines of constructive interference will
      not change position, but the interference
      will become more complete.

16. (1 pt) During your visit to the International
   Space Station you take along your 90 mm refracting telescope, which has high-quality, nearly perfect
   lenses. You point it at a star known to be a binary system (two stars close together), but the image that
   you see looks like a single star. You double the magnification, but it still looks the same. To resolve
   the two stars into separate images, you would have to
   a) replace the objective lens with one having a shorter focal length.
   b) reduce the aperture diameter by covering the objective lens with black paper with a relatively small
      hole cut in it.
   c) get a telescope with a larger diameter objective lens.
   d) use instead a reflecting telescope with a 90 mm diameter objective mirror.

17. (2 pts) The graph below shows the intensity pattern of interference from laser light shining through
    several narrow closely spaced slits and onto a screen. How many slits are there? The angle \( \delta \) is the
    phase difference, in radians, between the light from two adjacent slits (so that the big peaks are
    separated by \( 2\pi \) radians in phase).

   ![Intensity graph]

   a) Three slits.   b) Four slits.   c) Five slits.   d) Six slits.
18. (1 pt) Suppose that you are trying to resolve two closely spaced spectral lines using a spectrometer with a diffraction grating. You are looking at the 3rd-order lines of the diffraction/interference pattern, but unfortunately, the two lines are blurred together and cannot be resolved. Which of the following changes would increase the spectrometer resolving power and improve your ability to resolve the two lines?
   a) Increase the number of diffraction-grating lines that are illuminated by the light from the source being studied, but keep the same spacing between the lines.
   b) Decrease the distance between the diffraction grating and the telescope used to view the lines.
   c) Keep the number of illuminated diffraction-grating lines constant, but decrease the spacing between the lines.
   d) Look instead at the 1st-order lines.

19. (1 pt) A 1 cm diameter hole is drilled in the center of a 10 cm square plate of aluminum. When the aluminum is cooled down afterwards, the diameter of the hole will
   a) decrease   b) increase   c) remain unchanged

20. (1 pt) When a block of metal is heated up, the length, height, and width each increase by 1%. By how much does the volume of the block increase?
   a) 0.0001%   b) 1%   c) 2%   d) 3%

21. (1 pt) Several ice cubes are floating in a bucket of fresh water. When the ice melts, the water level in the bucket
   a) increases   b) decreases   c) remains unchanged.   d) increases.

22. (2 pts) A mechanic using a manual hydraulic jack to lift a car of weight 27,000 N pushes on a reservoir of oil using a piston of radius 1 cm. The car is lifted by the same oil pushing on a piston of radius 30 cm. With how much force $F$ must the mechanic push in order to lift the car slowly and steadily?
   a) 30 N   b) 300 N   c) 700 N   d) 13,500 N

23. (2 pts) In the previous problem, what distance must the mechanic push the small piston in order to lift the car 1 cm?
   a) 1 cm   b) 30 cm   c) 900 cm   d) 2700 cm

24. (2 pts) Two parallel steel plates $d=5$ mm apart have some high-viscosity oil between them. They are kept sliding past each other at constant speed by the application of a force to the top plate (the bottom plate is fixed in place to the floor). By what factor must the force be changed to keep the same speed if both the plate separation and the viscosity of the oil are doubled?
   a) 0.5
   b) 1.0 (no change)
   c) 2.0
   d) 4.0

25. (2 pts) A telescope lens is coated with a thin film to reduce the reflection of visible light of wavelength $\lambda$. If the index of refraction $n$ of the coating material is greater than the index of refraction of the lens glass, then the thickness of the coating should be
   a) $\lambda / (8n)$
   b) $3\lambda / (4n)$
   c) $\lambda / (4n)$
   d) $\lambda / (2n)$
26. (2 pts) Consider the following intensity pattern for interference of light from two slits. What is the ratio of the slit spacing \( d \) to the slit width \( a \)?
   a) \( d/a = 6 \)
   b) \( d/a = 5 \)
   c) \( d/a = 4 \)
   d) \( d/a = 3 \)

27. (10 pts) An insulated beaker with negligible mass contains 0.30 kg of water at a temperature of 60°C. How many kilograms of ice at a temperature of -20°C must be dropped into the water so that the final temperature of the system will be 40°C? See the front page for relevant numerical constants.

\[
C\text{onstant of energy} \\
\text{Heat gained by ice} = \text{heat lost by water} \\
m_i \cdot C_i (0 - (-20)) + m_f \cdot C_f + m_w \cdot C_w (40 - 0) = m_w \cdot C_w (60 - 40) \\
m_i = \frac{0.3 \cdot 4200 \cdot 20}{2100 \cdot 20 + 330,000 + 4200 \cdot 40} = \frac{25,200}{542,000} \text{ J/kg} \\
m_i = 0.047 \text{ kg} = 47 \text{ g}
\]

28. (10 pts) Vision correction:
   a) A far-sighted man (hyperopic) has a near point of 50 cm. What should be the focal length of the contact lenses used to correct his vision, such that his corrected near point will be 25 cm?

   \[
   \text{Object at } 25 \text{ cm} \Rightarrow \text{virtual image at } 50 \text{ cm} \\
   s = 25 \text{ cm} \\
   \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{25} - \frac{1}{50} = -\frac{1}{50} \\
   f = -50 \text{ cm}
   \]

   b) A near-sighted man (myopic) has a far point of 250 cm. What should be the focal length of the contact lenses used to correct his vision?

   \[
   \text{Object at } \infty \Rightarrow \text{virtual image at } 250 \text{ cm} \\
   s = \infty \\
   s' = -250 \text{ cm} \\
   \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{s'} = \frac{-1}{250 \text{ cm}} \\
   f = -250 \text{ cm}
   \]
29. (12 pts) Laser light is shining through 4 equally spaced slits, with distance \(d = 0.2\) mm between adjacent slits. The slit width is much less than \(d\), and the wavelength of the light is \(\lambda = 500\) nm.

a) Draw two phasor diagrams that illustrate how the equal amplitudes from the four slits add together at the locations of the first minimum (picture on left) and second minimum (picture on right) away from the central maximum in the interference pattern.

\[ \phi = \frac{\pi}{2} \]

b) What is the phase angle between phasors from adjacent slits in the case of the first minimum?

\[ \phi = \frac{\pi}{2} = 90^\circ \]

c) If a screen is placed 2 m away behind the slits, what is the distance in meters from the central maximum \((m=0)\) on the screen to the brightest point on the second principal maximum \((m=1)\)? You may use small-angle approximations: \(\theta \approx \sin \theta \approx \tan \theta\).

\[ d \sin \theta = \lambda \quad \text{so} \quad \theta \approx \frac{\lambda}{d} \quad (\phi = 360^\circ) \]

\[ L = 2 \text{ m} \]

\[ y = L \tan \theta \approx L \theta \approx L \frac{\lambda}{d} = 2.0 \frac{500 \cdot 10^{-9} \text{ m}}{2 \cdot 10^{-2} \text{ m}} = 5 \cdot 10^{-3} \text{ m} \]

\[ y = 0.50 \text{ cm} \]

d) What is the distance on the screen from the central maximum to the first minimum adjacent to the central peak?

\[ \phi = 90^\circ \] is \(\frac{1}{4}\) of \(360^\circ\)

\[ \text{So } y = \frac{1}{4}(0.5 \text{ cm}) = 0.125 \text{ cm} \]

\[ y = 0.13 \text{ cm} \]
30. (13 pnts) An architectural engineer desires a fountain in front of her latest building. She wants the water to squirt 20 m high from the narrow opening in the pipe illustrated here. Use \( g = 10 \text{ m/s}^2 \) in the following for the acceleration of gravity. Assume that friction and viscosity effects are negligible. The radius of the pipe is 10 cm, except at the nozzle (Point 2), where it necks down to a radius of 7.1 cm.

a) What must be the speed of the water just at Point 2, the exit to the pipe, in order to reach the desired altitude?

\[
\frac{1}{2} m v_z^2 = mgh
\]

\[
v_z = \sqrt{2gh} = \sqrt{2 \cdot 10 \cdot 20} = 20 \text{ m/s}
\]

b) What then is the speed of the water at Point 1, in the pipe 2 meters lower?

\[
\frac{V_1 A_1}{A_2} = \frac{v_z}{v_1} \quad \text{Continuity}
\]

\[
v_1 = \frac{A_2}{A_1} v_z = \left( \frac{r_2}{r_1} \right)^2 v_z = \left( \frac{7.1}{10} \right)^2 (20) = 10 \text{ m/s}
\]

c) What must be the gauge pressure of the water at Point 1? (Gauge pressure is the difference between the absolute pressure and atmospheric pressure.)

\[
p_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g \Delta y
\]

\[
\Delta p = p_1 - p_2 = \frac{1}{2} \rho (v_1^2 - v_2^2) + \rho g (2 \text{ m})
\]

\[
= \frac{1}{2} 1000 (400 - 100) + 1000 \cdot 10 (2)
\]

\[
= 150,000 + 20,000
\]

\[
\Delta p = 170,000 \text{ Pa}
\]

d) What is the flow rate of water from the fountain, in cubic meters per second?

\[
\frac{dV}{dt} = 5, A_1 = (10) \left( \pi \cdot 0.07^2 \right) = 0.16 \text{ m}^3/\text{s}
\]
31. (10 pts) A converging lens of 10 cm focal length is placed right at the center of curvature of a concave mirror with 10 cm radius of curvature, as illustrated below. The object is placed 5.0 cm from the lens. The two foci of the lens are labeled with f, and the center of curvature of the mirror is labeled with C.

a) Draw a ray diagram showing the formation of the image from the lens. Then use this image as an object for the mirror and draw rays showing the formation of the image from the mirror. (A third image is formed from the light passing through the lens a second time, but do not show it, or else the drawing will get too cluttered.)

b) Is the first image (from the lens) real or virtual?

c) Is the second image (from the mirror) real or virtual?

d) Calculate the position of the first image from the lens equation.

\[ s = 5.0 \text{ cm} \quad f = 10 \text{ cm} \]

\[ \frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{10} - \frac{1}{5} = -\frac{1}{10} \]

\[ s' = -10 \text{ cm} \]

e) Then calculate the position of the second image from the mirror equation.

\[ s = 10 + 10 = 20 \text{ cm} \]

\[ f = \frac{20}{5} = 4.0 \text{ cm} \]

\[ \frac{1}{s'} = \frac{1}{s} - \frac{1}{f} = \frac{15}{100} \]

\[ s' = 6.7 \text{ cm} \]

f) Will the third image (from the second passage of light through the lens) be real or virtual?
32. (10 pts) A 0.015 kg string 1.5 m long that is fixed at both ends and under a tension of 100 N is vibrating in its third harmonic. The maximum displacement of any segment of the string is 2.0 mm.

a) Graph the shape of the string when it is at its maximum amplitude of oscillation. (You do not need to draw a precise sine function, but do put the nodes and antinodes in the correct locations with the correct amplitude.)

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b) What is the wavelength $\lambda$ of this standing wave?

\[ \lambda = 1.0 \text{ m} \]

c) What is the frequency $f$ of the wave oscillation?

\[ \omega = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100 \text{ N}}{0.015 \text{ kg} / 1.5 \text{ m}}} = 100 \text{ m/s} \]

\[ f = \frac{\omega}{\lambda} = \frac{100 \text{ m/s}}{1.0 \text{ m}} = 100 \text{ Hz} \]

d) Write a wave function $y(x,t)$ appropriate for this standing wave.

\[ y(x,t) = A \cos \omega t \sin kx \]

\[ A = 2.0 \text{ mm} \]

\[ \omega = 2\pi f = 630 \text{ rad/sec} \]

\[ k = \frac{2\pi}{\lambda} = 6.3 \text{ m}^{-1} \]