## Final Exam

Print your name: ___________________________

<table>
<thead>
<tr>
<th>Problems</th>
<th>Maximum Score</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>5-13</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>14-19</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>20-25</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>26, 27</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>28, 29</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Closed book; no notes. Non-programmable calculators without alphanumeric keyboards are okay. A straightedge and pencil are needed for the ray diagram. Partial credit of 1 point may be given on multiple-choice questions in case your response is wrong if you clearly write down a relevant equation or concept. You may assume that any numerical constant provided in this exam is accurate to two significant figures.

The following constants and equations may or may not be needed:
- Acceleration of gravity: $g = 9.8 \text{ m/s}^2$
- Speed of sound in air: 350 m/s.
- Speed of light in vacuum: $3.0 \times 10^8 \text{ m/s}$.
- Gas constant: $R = 8.3 \text{ J/mol K}$
- Stefan-Boltzmann constant: $\sigma = 5.7 \times 10^{-8} \text{ W/(m}^2\text{K}^4)$
- Ratio of specific heat at constant pressure to that at constant volume, for air: $\gamma = 1.4$
- Young’s modulus of steel: $Y = 20 \times 10^9 \text{ Pa}$
- Density of water: 1000 kg/m$^3$
- Average power transmitted by a harmonic wave of amplitude $A$ and angular frequency $\omega$, on a string of tension $F$: $P = \frac{1}{2} \sqrt{\mu F \omega^2 A^2}$
- Intensity of a sound wave: $I = \frac{1}{2} \sqrt{\rho B} \omega^2 A^2 = \frac{P_{\text{max}}^2}{2\rho v} = \frac{P_{\text{max}}^2}{2\sqrt{\rho B}}$
- Relation between pressure and displacement amplitudes in a longitudinal sound wave: $P_{\text{max}} = B k A$.
- Wave equation: $\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} = 0$
- Doppler shift formula: $f_L = \frac{1 + v_L/v}{1 + v_S/v} f_S$.
- Angle of a shock wave: $\sin \alpha = v/v_s$
- Malus’ law: $I = I_{\text{max}} \cos^2 \phi$
- Equation for images formed by a single refracting surface: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$
- Lateral magnification of a single refracting surface: \( m = \frac{n_a s'}{n_b s} \)
- Thin lens equation: \( \frac{1}{s} + \frac{1}{s'} = (n - 1) \left[ \frac{1}{f_1} + \frac{1}{f_2} \right] \)
- Angular magnification of a magnifying glass: \( M = \frac{25 \text{ cm}}{f} \)
- Angular magnification of a telescope: \( M = -\frac{f_1}{f_2} \)
- Magnification of a compound microscope: \( M \approx \frac{1}{f_1 f_2} \)
- Intensity for a single-slit diffraction/interference pattern: \( I = I_0 \left( \frac{\sin[\pi a \sin \theta / \lambda]}{\pi a \sin \theta / \lambda} \right)^2 \)
- Intensity for a 2-slit interference pattern: \( I = I_0 \cos^2 \left( \frac{\phi}{2} \right) \left( \frac{\sin (\beta / 2)}{\beta / 2} \right) \)
  \[ \phi = \frac{2\pi l}{\lambda} \sin \theta \quad \text{and} \quad \beta = \frac{2\pi d}{\lambda} \sin \theta \]
- Bragg condition for constructive interference of x-rays reflected from a crystal: \( 2d \sin \theta = n\lambda \).
- Angular radius of the first dark ring of a diffraction/interference pattern from a circular aperture: \( \sin \theta_1 = 1.22 \frac{\lambda}{D} \).
- Bernoulli's equation: \( p_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2 \).
- Poiseuille's equation: \( \frac{dV}{dt} = -\frac{\pi r^4}{8} \left( \frac{p_1 - p_2}{L} \right) \frac{4\eta}{r} \)
- Stoke's law: \( F = 6\pi \eta r V \)

1. (2 pts) A toy boat of volume 0.005 m\(^3\) weighing 10 kg is placed into a bathtub filled to the brim with water. What is the mass of the water that flows over the edge of the bathtub?
   a) 1/10 kg. \[ m = \frac{10}{500} = 2.00 \cdot 10^{-2} \text{ kg} \]
   b) 5 kg.
   c) 10 kg.
   d) Not enough information is given to answer this question.

2. (2 pts) Fluid is flowing through a flow tube of varying radius, as shown in the figure. At point B the radius is \( \frac{1}{4} \) the value at point A. How do the velocities of the fluid compare at the two points?
   a) \( v_A = 2x v_B \)
   b) \( v_A = 4x v_B \)
   c) \( v_A = 16x v_B \)
   d) \( v_A = 2x v_B \)
   e) \( v_A = 4x v_B \)

3. (2 pts) A highly viscous liquid is being pumped through a long cylindrical pipe. In order to increase the flow rate by a factor of 16 without increasing the pressure, the diameter of the pipe should be increased by a factor of
   a) 2.
   b) 4.
   c) 16.
   d) 25.

4. (2 pts) The flow of a fluid through a pipe is found to be turbulent. Which of the following actions could result in a laminar flow, assuming all other properties are kept constant?
   a) Use a more viscous fluid.
   b) Increase the diameter of the pipe.
   c) Increase the velocity of the fluid flow.
   d) Use a more dense fluid.
5. (2 pts) Monochromatic light of wavelength $\lambda = 500 \text{ nm}$ is used to form an interference pattern using a diffraction grating of 10 slits. The angle between the central maximum and the first maximum to one side is found to be $10^\circ$. The grating is then exchanged for a second grating with the same slit spacing but with 20 slits. For the second grating, the angle between the central maximum and the first maximum to one side is:

a) $5^\circ$  
   b) $10^\circ$  
   c) $20^\circ$  
   d) $40^\circ$

6. (2 pts) A thin film is applied to the surface of a window in order to maximize the reflection of infrared light from the surface (while still allowing most of the visible light to pass through). The film has a thickness equal to $1/4$ the wavelength of the infrared light. The index of refraction of the film should be:

- less than that of the glass.
- equal to that of the glass.
- greater than that of the glass.
- the reciprocal of that of the glass.

7. (2 pts) Two identical containers of water have water filled to equal levels, but one container has three wooden balls floating in it, as illustrated below. How do the weights of the containers, including contents, compare? 
   a. Archimedes' principle, explained in lecture.
   b. The container of water with floating balls weighs more.
   c. The container of water with no floating balls weighs more.
   d. The two containers of water weigh the same.
   e. The result depends on whether the balls are more or less dense than water.

8. (2 pts) A toy boat holding an iron ball is floating in a bathtub. When the boat overturns and the iron ball sinks to the bottom of the tub (with the boat still floating), what happens to the water level in the tub?
   a. The water level rises.
   b. The water level decreases.
   c. The water level remains the same.

9. (2 pts) The present temperature of the cosmic background radiation, produced about 300,000 years after the Big Bang, is approximately
   a) 3 K  
   b) 100 K  
   c) 5800 K  
   d) 3,000,000 K

10. (2 pts) A metal ring is heated to a high temperature. As it heats up, does the hole in the center:
    a) grow larger?
    b) get smaller?
    c) stay the same size?

11. (2 pts) A physicist is using a diffraction grating to study the spectrum of an element. She sees a broad line that she suspects is really two lines merged together. In order to resolve the two lines, she should:
    a) increase the number of lines in the diffraction grating that are illuminated.
    b) increase the distance between the slits of the diffraction grating.
    c) make the light beam passing through the diffraction grating more narrow.
    d) move the screen farther away from the diffraction grating.

12. (2 pts) In a harmonic sound wave, the points where the molecular displacement is zero are also
    a) the points where the pressure is equal to the equilibrium pressure.
    b) the points where the pressure deviation from equilibrium is maximum.
    c) the points where the molecular speed is zero.
    d) the points where the density of the gas is minimum.

13. (2 pts) If you triple the temperature of an object that is much hotter than its surroundings, it will radiate energy
    a) 3 times faster.
    b) 9 times faster.
    c) 27 times faster.
    d) 81 times faster.

\[ H \propto T^4 \]
14. (2 pts) A mechanic using a manual hydraulic jack to lift a car of weight 20,000 N pushes on a reservoir of oil using a piston of radius 3 cm. The car is lifted by the same oil pushing on a piston of radius 30 cm. With how much force \( F \) must the mechanic push in order to lift the car slowly and steadily?

- \( a)\) 100 N
- \( b)\) 200 N
- \( c)\) 667 N
- \( d)\) 2000 N

15. (2 pts) Consider the following intensity pattern for interference of light from two slits. What is the ratio of the slit spacing \( d \) to the slit width \( a \)?

- \( a)\) 3
- \( b)\) 4
- \( c)\) 5
- \( d)\) 6

16. (2 pts) To use a magnifying glass effectively, the object should be placed on the side of the lens opposite the eye and

- \( a)\) slightly further from the lens than the lens focal length
- \( b)\) as close to the lens as possible
- \( c)\) slightly closer to the lens than the lens focal length.
- \( d)\) as far away as possible.

17. (2 pts) Imagine holding a perfectly spherical ball bearing in a beam of light. The ball casts a shadow on a distant screen. The exact center of the shadow is

- \( a)\) a small bright spot.
- \( b)\) darker than the rest of the shadow.
- \( c)\) bright or dark, depending on the distance between the sphere and the screen.
- \( d)\) the same darkness as the rest of the shadow, but slightly less dark than if there were no diffraction.

18. (2 pts) Two parallel steel plates \( d=5 \text{ mm} \) apart have some high-viscosity oil between them. They are kept sliding past each other at constant speed by the application of a force to the top plate. By what factor must the force be changed to keep the same speed if both the plate separation and the surface area \( A \) of the top plate are doubled?

- \( a)\) 0.5
- \( b)\) 1.0 (no change)
- \( c)\) 2.0
- \( d)\) 4.0

19. (2 pts) To see light from galaxies as far away in the universe as possible, it is most important for the astronomical telescope to have

- \( a)\) the highest possible magnification.
- \( b)\) the largest possible aperture of the objective mirror or lens.
- \( c)\) the lowest possible chromatic distortion.
- \( d)\) the longest possible focal length of the objective mirror or lens.
20. (2 pts) A perfectly exposed photograph is taken with a 1/125 s long exposure using a 50 mm focal length lens and an f-number of 1.4. The lens is then replaced by a 135 mm focal length lens with an f-number of 2.8. How long should the exposure be in order to photograph correctly the same subject with the new lens?
   a) 1/4 s.  
   b) 1/30 s.  
   c) 1/60 s.  
   d) 1/250 s.

21. (2 pts) The American scientists Albert Michelson and Edward Morley used the first Michelson interferometer to
   a) measure diffraction of x-rays from crystals.
   b) measure the speed at which Earth travels through the "ether."
   c) measure the size of atoms.
   d) produce the first holograms.

22. (2 pts) In the case of a nearsighted person (myopic) the image of a very distant object
   a) forms between the lens and the retina.
   b) would form behind the retina, on the opposite side from the lens.
   c) focuses in a different plane for horizontal versus vertical lines.

23. (2 pts) To obtain corrected vision, a farsighted person (hyperopic) should wear glasses with
   a) diverging lenses.
   b) converging lenses.

24. (2 pts) A string of length $L$ is vibrating in its fourth harmonic with frequency $f$. Which of the following wave functions correctly represents the wave on the string?
   a) $y(x,t) = A \cos(2\pi f \cdot t) \sin \left( \frac{4\pi x}{L} \right)$
   b) $y(x,t) = A \cos(2\pi f \cdot t) \sin \left( \frac{2\pi x}{L} \right)$
   c) $y(x,t) = A \cos \left( \frac{2\pi}{L} \left(x - Lf \cdot t\right) \right)$
   d) $y(x,t) = A \sin \left( \frac{2\pi f \cdot t - 4\pi x}{L} \right)$

25. (2 pts) The graph below shows the intensity pattern of interference from laser light shining through several narrow slits and onto a screen. How many slits are there? The angle $\delta$ is the phase difference, in radians, between the light from two adjacent slits (the big peaks are separated by $2\pi$ radians in phase).

```
\begin{center}
\includegraphics[width=0.5\textwidth]{interference_graph.png}
\end{center}
```

a) Two slits.  
b) Three slits.  
(c) Five slits.  
(d) Six slits.
26. Consider an interference pattern formed by light passing through 5 identical, equally-space slits.
   a) (2 pts) What is the phase angle between waves from adjacent slits at the first minimum (away from the central maximum) on a distant screen?

\[ \phi = \frac{360^\circ}{5} = 72^\circ \]

b) (2 pts) Draw a phasor diagram illustrating how the five wave functions add together at the first minimum.

\[ \text{regular pentagon} \]

\[ \text{4 phases cancel} \]

\[ \frac{1}{5^2} I_0 = \frac{1}{25} I_0 \]

c) (2 pts) If the intensity at the peak of the central maximum is \( I_0 \), then what is the intensity at the point on the screen where the phase angle between waves from adjacent slits is 180°?

\[ \phi = 180^\circ \]

27. (10 pts) Under standard conditions, the density of air is 1.3 kg/m³ and that of helium is 0.18 kg/m³. A helium balloon lifts a basket and cargo of total mass 200 kg (including the mass of the empty balloon). What must be the volume of the balloon in order that it floats in the air at constant altitude?

Buoyant force = weight

\[ \text{air } V g = (200 \text{ kg})g + \rho_{\text{He}} V g \]

\[ (\text{air} - \rho_{\text{He}}) V = 200 \text{ kg} \]

\[ V = \frac{200 \text{ kg}}{(1.3 - 0.18) \text{ kg/m}^3} = 180 \text{ m}^3 \]
28. (8 pts) Two rectangular pieces of plane glass are laid one upon the other on a table. A strip of paper is placed between them at one edge so that a very thin wedge of air is formed. The plates are illuminated at normal incidence by light from a sodium lamp ($\lambda=590$ nm). Interference fringes are formed, with 16 fringes per centimeter. Find the angle of the wedge.

\[ \text{Distance between fringes} = \frac{1}{16} \text{cm} = 0.0625 \text{cm} \]

\[ \tan \alpha = \frac{\frac{1}{2} \lambda}{0.0625 \text{cm}} = \frac{1}{2} \frac{590 \times 10^{-9}}{0.0625 \times 10^{-2}} \]

\[ \tan \alpha = 4.72 \times 10^{-4} \]

\[ \alpha = 0.027^\circ \]

29. A vertical cylindrical pipe of 5.0 cm radius and 1.5 m length is filled with water. A speaker emitting sound at a frequency of $f=660$ Hz is mounted just above the open top of the tube. Water is drained from the bottom until the observers in the room hear a maximum sound intensity (resonance). This water level, 22 cm below the top of the pipe, is marked. Then, more water is drained out until a second resonance is heard. The difference in water level between the two resonances is 25 cm. (I did this demonstration in lecture.)

a) (2 pts) What is the wavelength of the sound in the pipe?

\[ 25 \text{cm} = \frac{1}{2} \lambda \]

\[ \lambda = 50 \text{cm} \]

b) (2 pts) What is the speed of the sound in the pipe?

\[ v = f\lambda = (660 \text{ Hz})(0.50 \text{m}) = 330 \text{m/s} \]

c) (2 pts) How far above the top of the tube is the closest displacement antinode?

\[ \frac{1}{4} \lambda = 12 \text{ cm} = 0.5 \text{ cm} \]

I will accept any answer, since my typo messed this up.
30. A venturi flowmeter is used to measure the speed of flow of a fluid through a pipe. The radius of the pipe is 15 cm. At one point the radius is reduced to 5.0 cm. One vertical tube sticks out of the top of the pipe in the narrow section and another in the wide section. Both tubes are open at the top, and the difference in height between the water in the two tubes is 3.0 m. (The picture below is not to scale.)

![Diagram of venturi flowmeter with dimensions and pressure and velocity calculations]

a) (3 pmts) What is the difference in pressure between the large and small sections of pipe?

\[ P_A - P_B = \rho g h = 1000 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times 3 \text{ m} = 2.94 \times 10^4 \text{ Pa} \]

b) (5 pmts) What is the velocity of the water in the wide section of the pipe?

**Continuity:** \[ U_A A_A = U_B A_B \Rightarrow U_B = \left( \frac{r_A}{r_B} \right)^2 U_A \]

**Bernoulli:** \[ P_A + \frac{1}{2} \rho U_A^2 = P_B + \frac{1}{2} \rho U_B^2 \]

\[ U_B^2 - U_A^2 = \frac{2(P_A - P_B)}{\rho} \Rightarrow U_A = \sqrt{\frac{2(P_A - P_B)}{(r_A/r_B)^4 - 1}} = \frac{2(P_A - P_B)}{\rho} \]

\[ U_A = \sqrt{\frac{2(P_A - P_B)}{(0.15 m/0.05 m)^4 - 1}} = \frac{2(2.94 \times 10^4)}{1000 \times 3^2 - 1} = 0.86 \text{ m/s} \]

c) (2 pmts) What volume of water flows through the pipe per second?

\[ \frac{dV}{dt} = \pi A U = \pi (0.15 \text{ m})^2 0.86 \text{ m/s} = 0.061 \text{ m}^3/\text{s} \]
31. A diverging lens with focal length \( f_1 = -10 \text{ cm} \) and a converging lens of focal length \( f_2 = 10 \text{ cm} \) are separated by 15 cm. An object 5.0 cm tall is placed 10 cm from the diverging lens, on the side opposite the other lens.

a) (3 pts) Draw a ray diagram showing the formation of the images from the first and second lenses.

b) (2 pts) Calculate the image distance of the intermediate image from the first lens.

\[
\frac{1}{s'_1} = -\frac{1}{f_1} - \frac{1}{s} = -\frac{1}{-10} - \frac{1}{10} = -\frac{1}{5}
\]

\[s'_1 = -5 \text{ cm}\]

c) (1 pt) Is this intermediate image real or virtual? 

d) (2 pts) Calculate the size of the image from the first lens.

\[
y'_1 = -\frac{s'_1}{s} y = -\frac{-5}{10} y = \frac{1}{2} y = 2.5 \text{ cm}
\]

e) (2 pts) Calculate the image distance of the final image from the second lens.

\[
s_2 = 15 \text{ cm} + 5 \text{ cm} = 20 \text{ cm}
\]

\[
\frac{1}{s'} = \frac{1}{f_2} - \frac{1}{s_2} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}
\]

\[s' = 2 \text{ cm}\]