Physics 6C
Introduction to Physics III
Electricity and Magnetism

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**AC Current and Voltage**

\[ V(t) = V_0 \cos \omega t \]

mean voltage is zero!

\[ V(t)^2 = V_0^2 \frac{1}{2} (1 + \cos 2\omega t) \]

The rms voltage is

\[ V_{\text{rms}} \equiv \sqrt{\langle V^2 \rangle} = \frac{V_0}{\sqrt{2}} \]
Represent the cosine function as the $x$-component of a rotating vector. This provides a graphical means to keep track of both amplitude and phase.

(Note for the mathematically inclined: this is equivalent to representing the cosine function as the real part of a complex number.)
AC Circuits: Resistance

\[ V(t) - IR = 0 \quad \text{Kirchhoff’s rule} \]

\[ I(t) = \frac{V(t)}{R} \]

\[ I(t) = \frac{V_0}{R} \cos \omega t \]

Energy is dissipated (turned into heat) by the resistor!

\[ P = I^2 R = \frac{V_0^2}{R} (\cos \omega t)^2 \]

\[ P_{av} = \frac{1}{2} \frac{V_0^2}{R} = \frac{V_{rms}^2}{R} \]
Phasors for Resistive Circuit

The resistor voltage and current oscillate in phase.

Voltage phasor, length $V_R$

Current phasor, length $I_R$

Instantaneous current and voltage
AC Circuits: Capacitance

Kirchhoff’s rule

\[ V(t) - \frac{Q}{C} = 0 \]

\[ \frac{dV}{dt} = \frac{1}{C} \frac{dQ}{dt} = \frac{1}{C} I \]

\[ I(t) = C \frac{dV}{dt} = -C\omega V_0 \sin \omega t \]

\[ I(t) = \frac{V_0}{1/\omega C} \cdot \cos(\omega t + \frac{\pi}{2}) \]

Capacitive “reactance”:

\[ X_C \equiv \frac{1}{\omega C} \]

\[ I_{\text{max}} = \frac{V_0}{X_C} \]

Energy is stored, NOT dissipated!
Phasors for Capacitive Circuit

(a) $i_C$ peaks $\frac{1}{4}T$ before $v_C$ peaks. We say that the current leads the voltage by 90°.

(b) The current phasor leads the voltage phasor by 90°.

These are the instantaneous current and voltage.
Exercise 35-9

- A 20 nF capacitor is connected across an AC generator that produces a peak voltage of 5.0 V.
  
  a) At what frequency $f$ is the peak current 50 mA?
  
  b) What is the instantaneous value of the emf at the instant when $i_C = I_C$?

\[ v(t) = 5.0 \cos(2\pi ft) \]

Notation: our textbook uses $i$ to represent the time-dependent current and $I$ to represent the amplitude:

\[ i_C \equiv I(t) = I_C \cos(\omega t + \frac{\pi}{2}) \]
Kirchhoff’s loop rule holds at every instant in time:

\[ \mathcal{E}(t) = v_R(t) + v_C(t) \]

We want to find the resulting current, which generally will not be in phase with the voltage:

\[ i(t) = I_{\text{max}} \cos(\omega t + \phi) \]

We have to find both the amplitude \( I_{\text{max}} \) and the phase \( \phi \).

\[ v_R = I_{\text{max}} R \cos(\omega t + \phi) \quad \text{The resistor voltage is in phase with current} \]

\[ v_C = I_{\text{max}} \frac{1}{\omega C} \cos(\omega t + \phi - \frac{\pi}{2}) \quad \text{The capacitor voltage lags behind the current by 90 degrees.} \]
RC Circuit

\[ \mathcal{E}(t) = v_R(t) + v_C(t) \]

\[ \mathcal{E}_0 \cos \omega t = I_{\text{max}} R \cos(\omega t + \phi) + I_{\text{max}} \frac{1}{\omega C} \cos(\omega t + \phi - \frac{\pi}{2}) \]

This equation can be solved for both \( I_{\text{max}} \) and \( \phi \) by using trig identities, but it is easier to do it graphically using phasors. The algebra then just looks like vector addition.

\[ \mathcal{E}_0 = \sqrt{V_R^2 + V_C^2} = I_{\text{max}} \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \]

\[ \tan \phi = \frac{V_C}{V_R} = \frac{1}{\omega RC} \]

As in a circuit with just a capacitor, the voltage lags behind the current, but by less than 90 degrees.