LECTURE #2

ON THE MENU:

1. UNDERSTAND \( \mathcal{E}_T \) ?
   BUT NOT LONG-TERM EVOLUTION...

2. DECOUPLING \( \to \) DIRECT DETECTION

[Probably next lecture]

\( \text{But some conjugated process in early universe: Kinetic Decoupling} \)

\( \text{Sets small-scale cut-off to matter power spectrum} \)

1. \( \mathcal{E}_T \): Starting point: Boltzmann equation

\[ \dot{f} + \nabla \cdot (f \mathbf{v}) = \sum_{i} \{ f_i \} - \{ f \} \]

\[ f = f(x, \mathbf{p}, t) \]

\( \text{Lagrangian\, Op: Changes in phase space density} \)

\( \text{Collision\, Op: Changes in phase space density} \)

\[ \text{In FRW cosmologies (homogeneous, isotropic)} \, f_\ast f(E, t) \]
WE WANT TO CALCULATE NUMBER DENSITIES:

\[ n(t) = \sum_{\text{spin}} \int \frac{d^3p}{(2\pi)^3} f(E, t) \]

... INTERESTING DERIVATION OF RESULTING EQ FOR \( n(t) \):

GONDINO + GELMINI NPB 360 (1991) 145

\[ 1+2 \leftrightarrow 3+4 \]

\[ \chi \chi \underline{\rightarrow} \text{ IN PA. EQ.} \]

\[ \int C[f] \frac{d^3p}{(2\pi)^3} \rightarrow \frac{d\theta}{d^3q} + \int \frac{d^3p}{(2\pi)^3} \]

\[ \langle \gamma M^{\mu\nu} \rangle \left( h_1 h_2 - h_1^e h_2^e \right) \]

(INTEGRATED OVER MOMENTUM)

\[ \sum_{f} \sum_{1 \rightarrow f} \]

WHERE:

\[ \gamma M^{\mu\nu} = \frac{\gamma A_{eff}^2 \gamma_{12} \gamma_{12}}{E_1 E_2} \]

\[ \gamma_{12} \gamma_{12} \text{ Lorentz-invariant cross section} \]

(b). \( \gamma M^{\mu\nu} \rightarrow \gamma_{\text{rel}} = (\mathbf{v}_1 - \mathbf{v}_2) \)

IN REST FRAME OF 1 (C.O.M.)

\[ \mathbf{v}_1 = \frac{\mathbf{p}_1}{E_1} \]

(b). THERMAL AVERAGE

\[ \langle \gamma \rangle = \frac{\int \gamma M^{\mu\nu} e^{-\frac{E_1}{T}} e^{-\frac{E_2}{T}} d^3p_1 d^3p_2}{\int e^{-\frac{E_1}{T}} e^{-\frac{E_2}{T}} d^3p_1 d^3p_2} \]

MEANING OF VARIOUS TERMS
Show the denominator is, for \( m_1 = m_2 = m \):

\[
(4\pi m^2 T \, k_2 \left( \frac{m}{T} \right))^2
\]

Modified Bessel function of 2nd order.

\[
\text{Numerator: } \int_{m_1}^{\infty} \left( s - 4 m^2 \right) \sqrt{s} \, k_1 \left( \frac{\sqrt{s}}{T} \right) \, ds
\]

\( 4 m^2 \) Cross Section

\( \sqrt{s} \) Thermal Kernel

Key to understand important caveats to, e.g., \( \Omega_{\rm DM} \sim 8 \times 10^{-5} \).

Classic paper: Griest & Seckel, PRD 43 (1991) 3191

("3 exceptions in the calculation of relic abundances")

1. Resonances
2. Thresholds
3. Communication
   - All 3 exceptions relevant to SUSY DM!
1. **Resonances** *(Made Popular by "Funnel Region of MSSM/CMSSM")*

\[ \chi \rightarrow A, H \rightarrow F \]

\[ S = m_A^2 \rightarrow S - 4m_A^2 \]

\[ \text{e.g.: } m_A^2 \approx 4m_X^2 \]

So \( \langle J_U \rangle (T = 0) \) relevant for indirect detection can be much smaller than \( \langle J_U \rangle (T = T_{eq}) \).

Opposite can also be true: suppose \( m_A^2 \ll 4m_X^2 \)

\[ \text{Effect of resonance can be negligible at } T_{eq} \] (especially if \( \Gamma_A \ll T_{eq} \)) but dramatic at \( T = 0 \)!

\[ \Rightarrow \langle J_U \rangle_{T = 0} \ll \langle J_U \rangle_{T_{eq}} \]

But one more caveat: e.g. in SUSY

\( \tilde{\chi}_1 \) are Majorana: at \( T = 0 \) (s-wave)

\[ \chi \rightarrow H \rightarrow FP \] vanishes if \( h \) is CP even!

\[ \text{and CP-odd!} \]
2. Thresholds: Obvious Effect

\[
e.g. \frac{M_{\nu}}{\chi_1} \leq M_\nu
\]

\[
S = 4M_\nu^2
\]

Always produces \( \langle \nu \nu \rangle_{\text{f.o.}} > \langle \nu \nu \rangle_0 \)

3. Coannihilations

Relevant for "coupled" decoupling processes,

If \( e.g. M_2 - m_1 \lesssim T_{\text{f.o.}} \) (Otherwise \( Z \) dies by Boltzmann suppression)

\[
\langle \nu \nu \rangle \rightarrow \langle \nu_{\text{eff}} \nu \rangle = \sum_{ij} \xi_{ij} e^{-\frac{\Delta m_{ij}}{T}}
\]

\[
\sum_{i=1}^{N} g_i e^{-\frac{\Delta m_i}{T}}
\]

Counts the effect of the additional degree of freedom, suitably weighted.

Two Scenarios

1. "Parasite" Coannihilation (Denominator Dominates)

2. "Symbiotic" Coannihilation (Numerator Dominates)
1. UED \[ \Rightarrow \text{Compressed Spectrum,} \]

Ton of Coupled D.O.F., Overall Gives Suppression Effect

2. Stau (Chargino/Stop) Annihilation in MSSM

\[ \tilde{c}_i \rightarrow \tilde{c}_j + \tilde{c}_k \]

Low # Extra D.O.F., but EM Interacting

So far: CAVERTS TO LHS IN \[ \frac{1}{\sqrt{\text{MSSM}}} = H \]

What happens if \( H \) is changed?

Good Example: Quintessence Field (Salati, 2003)

\[ \phi - \text{Spatially Homogeneous Scalar, IR Field} \]

\[ P_\phi = \frac{1}{2} \left( \frac{d\phi}{dr} \right)^2 + V(\phi) \rightarrow \text{e.g.: } M_P^4 \exp \left( \frac{1}{M_P} \right) \]

\[ P_\phi = \frac{1}{2} \left( \frac{d\phi}{dt} \right)^2 - V(\phi) \]

So \( P_\phi = \omega P_\phi \) yields \( \omega = \begin{cases} \frac{1}{\omega} & \text{"Kination"} \\ \frac{1}{\omega} & \text{"Cosmologische"} \end{cases} \)
A cross-symmetric process to pair-annihilation is scattering off of SM particles.

- In "Late Universe": \( \bar{X}_9 \rightarrow X_9 \) 
  - e.g. quark in low-BCRG env. effectively: scattering off of a nucleus

- In "Early Universe", \( \bar{X}_f \rightarrow X_f \)
  - is process that keeps \( X \) in kinetic equilibrium

Kinetic decoupling signals start of (bottom-up) formation of structure, i.e. cut-off to smallest dark matter halos → important for cosmic simulations, also indirect DM detection ("boost factor")

Start with direct detection: what are energy scales?

\[ \sqrt{\sigma} \sim 200 \text{ km/s} \]

So: deeply non-relativistic! \[ \frac{\sqrt{\sigma}}{c} \sim 10^{-3} \]

Typical momentum transfer: standard scattering theory:

\[ \frac{1}{2} \mathbf{p}^2 = \frac{1}{2} m_{\text{red}}^2 v^2 (1 - \cos^2 \theta) \]

\[ m_{\text{red}} = \frac{m_X m_N}{m_X + m_N} \]

Typical energy transfer: \[ \Delta E = \frac{1}{2} \mathbf{p}^2 (2 m_N) = \frac{m_{\text{red}}^2 v^2}{m_X} (1 - \cos^2 \theta) \]
For WIMP in 10 - 500 GeV range hitting a nucleus with $m \sim 1 - 200$ GeV, $Q \sim 1 - 100$ keV

How does one detect this energy deposition?

Ionization on solids

Xenon, ZEPLIN

CDMS, EDWINS

Ionization in scintillators (detecting $\gamma$-s)

CRESST II

Temperature increase (detect phonons)

How do we calculate event rate for a given DM theory?

It's a multi-layered problem

Matrix elements of quark, gluons in a nucleon state

Quark - Gluon

Effective Lagrangian, e.g. $L = \bar{q} \gamma\nu q \bar{q} \gamma^\mu q + \ldots$
1. **Quark-Gluon**

Effective NR Lagrangian from Microscopic Theory

\[ \mathcal{L} = f_\bar{q} \bar{q} \gamma^\mu q \rho_\mu + \ldots \]

\[ f_\bar{q} \sim \frac{1}{m_\bar{q}} \]

2. **Nucleons \((p,n)\)**

Use matrix elements of quarks and gluons in a nucleon state (from XPT, scattering, etc...)

**E.g.**:

- **Scalar Interaction**
  \[ \langle n | m_\bar{q} \bar{q} | h \rangle = h \ h \ f_\bar{q} \]

  From HEAD of non-nucleon sigma term using XPT

- **Axial Current**
  \[ \langle n | \bar{q} \gamma_\mu \gamma_5 q | h \rangle = 2 \sum_\mu \Delta q \]

  From lepton-nucleon scattering

3. **Nucleus**

Nuclear wave functions \( \rightarrow \) add spin and scalar comp. of nucleons coherently to give WIMP-nucleus interaction
Sandwich nucleon DP's in a nuclear state

→ form factor suppression (coherence loss)

Spin-spin: lengthy...

Scalar: [nucleon DP has no spin structure]
  → only x nucleon number

Form factor at 0, simply Fourier transform of (non-zero mom. transfer) nucleon density.

\[ F(q) = \exp\left(-\frac{q}{2Q_0}\right) \]

\[ Q_0 = \frac{1.5}{m_N R_0^2} \quad \text{Nuclear coherence energy} \]

\[ R_0 \sim 10^{-13} \text{ cm} \left[ 0.7 + 0.91 \left(\frac{m_N}{6m}\right)^{1/3} \right] \]

Nuclear radius

What is the detection rate?

Per unit detector mass, \( R \sim \frac{\eta_x \cdot T \cdot \langle v \rangle}{m_N} \)

\[ \eta_x = \frac{S_{DM}}{m_x} \]

\[ dR = \frac{S_{DM}}{m_x m_N} \cdot V f(v) dv \cdot \left(\frac{d\Gamma}{d|q|^2}\right) d|q|^2 \]
CHANGING VARIABLES TO ENERGY DEPOSITION $Q$

$$\frac{dR}{d\alpha} = \frac{S_0 \rho_{dm}}{m \times m_{\text{red}}} \int_{\text{min}}^{\alpha} f(Q) \frac{f(v)}{v} dv$$

where $f(Q)$ encodes nuclear physics form factor.

$$\Gamma_0 \sim (Z f_p + (A-Z)f_n)^2$$

$$\frac{f_p}{m_p} = \sum_{q, u d s} \frac{f_{qg}}{m_{qg}} \left[ f_q + ... \right] + ...$$

**Two Remarks:**

1) $f_p$ usually $\sim f_n$ (exception: isospin violating D.M. !)

so $\Gamma_0 \sim A^2$ \rightarrow **Big Nuclei**

2) $f_q$ for SUSY NEUTRINOS:

\[ \begin{array}{c}
\text{#1} \\
\text{#2} \end{array} \]

Usually $\#1 < \#2$

#2 depends on gaugino-higgsino mixing (vanishes for pure higgsinos)
For spin dependent:

\[ \text{and again } #1' \subset #2' \]

\[ #2' \text{ depends only on Higgsino content.} \]

Many tools have dedicated calculations of:

- \( Q_x \), \( T_f \), etc.
- e.g. DarkSusy
- Micromegas
  (compatible with generic compiled input files)

Now: what happens for early universe scattering?