Homework Set #1.

Due Date: Monday October 16, 2012

Solve the following three exercises:

1. A particle starts at rest and moves along a cycloid whose equation is

   \[ x = \pm \left[ a \cos^{-1}\left( \frac{a-y}{a} \right) + \sqrt{2ay - y^2} \right]. \]

   There is a gravitational field of strength \( g \) in the negative \( y \) direction. Obtain and solve the equations of motion. Show that no matter where on the cycloid the particle starts out at time \( t = 0 \), it will reach the bottom at the same time.

2. A point particle of mass \( m \) is constrained to move frictionlessly on the inside surface of a circular wire hoop of radius \( r \), uniform density and mass \( M \). The hoop is constrained to the \( xy \)-plane, it can roll on a fixed line (the \( x \)-axis), but it does not slide, nor can it lose contact with the \( x \)-axis. The point particle is acted on by gravity exerting a force along the negative \( y \)-axis. At \( t = 0 \) suppose the hoop is at rest. At this time the particle is at the top of the hoop, and is given a velocity \( v_0 \) along the \( x \)-axis. What is the velocity \( v_f \), with respect to the fixed axis, when the particle comes to the bottom of the hoop? Simplify your answer in the limits \( m/M \to 0 \) and \( M/m \to 0 \).

3. A double plane pendulum consists of a simple pendulum (mass \( m_1 \), length \( l_1 \)) with another simple pendulum (mass \( m_2 \), length \( l_2 \)) suspended from \( m_1 \), both constrained to move in the same vertical plane.

   (a) Describe the configuration manifold \( Q \) of this dynamical system. Say what you can about \( TQ \) (the tangent bundle of \( Q \)).

   (b) Write down the Lagrangian of this system in suitable coordinates.

   (c) Derive Lagrange’s equations.

4. A cartesian coordinate system with axes \( x, y, z \) is rotating relative to an inertial frame with constant angular velocity \( \omega \) about the \( z \)-axis. A particle of mass \( m \) moves under a force whose potential is \( V(x, y, z) \). Set up the Lagrange equations of motion in the coordinate system \( x, y, z \). Show that these equations are the same as those for a particle in a fixed coordinate system acted on by the force \( -\nabla V \) and a force derivable from a velocity-dependent potential \( U \), and find \( U \).