1. My son Alex and I went to the park the other day, and Alex insisted on going on the swing. The swing has negligible mass, and is suspended by a rope of length \( l \). Assume the dimensions of Alex are negligible compared with \( l \), and that his mass be \( m \).

I pulled Alex back until the rope made an angle of exactly one radian with the vertical, then pushed him with a force \( F = mg \) along the arc of a circle (i.e. tangential to Alex’s direction of motion) until the rope was vertical, and, finally, I released the swing.

For what length of time did I push the swing? [You can assume that it is sufficiently accurate for this problem to approximate \( \sin \theta \approx \theta \) for \( \theta < 1 \)]

2. (a) Show that the moment of inertia of a thin rod about its center of mass is \( ml^2/12 \).

(b) A long thin tube of negligible mass is pivoted so that it may rotate without friction in a horizontal plane. A thin rod of mass \( M \) and length \( l \) slides without friction in the tube. Choose a suitable set of coordinates and write Lagrange’s equations for this system.

(c) Initially the rod is centered over the pivot and the tube is rotating with angular velocity \( \omega_0 \). Show that the rod is unstable in this position, and describe its subsequent motion if it is disturbed slightly.

(d) What are the asymptotic radial and angular velocities of the rod after a long time? [Assume the tube is long enough that the rod is still inside]

3. Two stars with masses \( m \) and \( M \) separated by a distance \( d \) are in circular orbits around the stationary center of mass. The stars may be approximated by point masses. In a supernova explosion, the star of mass \( M \) loses a mass \( \Delta M \). The explosion is instantaneous, spherically symmetric, and exerts no reaction force on the remnant. It also has no direct effect on the other star. Show that the remaining binary system is bound when \( \Delta M < \frac{M+m}{2} \).