Proton Decay

There are theoretical reasons to believe that the decay
\[ p \rightarrow e^+ + \pi^0 \]
might occur. The decay has never been observed, and this has been turned into an experimental limit on the proton lifetime:
\[ \tau_p = \frac{\hbar}{\Gamma} > 1.6 \times 10^{33} \text{ yr}. \]

The most general matrix element for the process, compatible with Lorentz invariance and momentum conservation, has the form:
\[ M = \bar{u}(\vec{p}_{e^+}) T u(\vec{p}_p), \quad \text{with} \quad T = A + B\gamma^5. \]

where A and B are constant.

(i) Which constraints would parity conservation impose on the constants A and B?

(ii) Calculate the decay width for the proton decay process as a function of A and B.

(iii) Calculate the constraints on A and B from the lower limit on the proton lifetime.

(iv) The \( \pi^0 \) is not directly observed, since it decays immediately into two photons. Knowing that the \( \pi^0 \) has spin zero, calculate the distribution of the angle the photons form with the direction of motion of the positron in the final state.