Homework Set #3.

Due Date - Oral Presentation: Wednesday October 21, 2015
Due Date - Written Solutions: Wednesday October 28, 2015

1. Muon decay in the Fermi theory
   A muon decays to an electron, an electron (anti)neutrino and a muon neutrino,
   \[ \mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e. \]
   In the so-called Fermi theory, the matrix element for this process, ignoring the electron and neutrinos masses, is given by
   \[ |M|^2 = 32 G_F^2 (m^2 - 2mE) mE, \]
   where \( m \) is the muon mass, \( E \) is the energy of the outgoing electron antineutrino, and \( G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2} \) is the Fermi constant.

   (a) Perform the integral over \( d\Pi_{\text{LIPS}} \) and show that the decay rate reads:
   \[ \Gamma = \frac{G_F^2 m^5}{192\pi^3}; \]
   (b) Convert from natural units to inverse seconds, using \( m = 106 \text{ MeV} \), and compare your result to the observed value \( \tau = \Gamma^{-1} = 2.20 \mu\text{s} \). How big is the discrepancy as a percentage? What might account for the discrepancy?

2. Mandelstam variables
   We calculated that the \( e^+ e^- \rightarrow \mu^+ \mu^- \) cross section had the form, in the CM frame,
   \[ \frac{d\sigma}{d\Omega} = \frac{e^4}{64\pi^2 E_{\text{CM}}^2} (1 + \cos^2 \theta). \]

   (a) Work out the Lorentz-invariant quantities
   \[ s = (p_{e^+} + p_{e^-})^2, \quad t = (p_{\mu^-} - p_{e^-})^2, \quad u = (p_{\mu^+} - p_{e^-})^2, \]
known as Mandelstam variables, in terms of $E_{\text{CM}}$ and $\cos \theta$ (still assuming $m_\mu = m_e = 0$).

(b) Derive a relationship between $s$, $t$ and $u$.

(c) Rewrite $\frac{d\sigma}{dt}$ in terms of $s$, $t$ and $u$.

(d) Now assume $m_\mu$ and $m_e$ are non-zero. Derive a relationship between $s$, $t$ and $u$ and the masses.