1.) Taylor expanding about the point $x = 0$, derive the binomial expansion

$$(1 + x)^p = 1 + p \cdot x + [p(p - 1)/2!] \cdot x^2 + [p(p - 1)(p - 2)/3!] \cdot x^3 + \ldots$$

(Note that this will only converge if $|x| < 1$.) For $p$ a positive integer, we can instead use the laws of algebra to derive an expression with a finite number $(p + 1)$ of terms. Is this consistent with the infinite series derived above from the Taylor expansion? Why or why not?

2.) Use the binomial expansion derived above to first order in $x$ to estimate the value of

$$\frac{1}{\sqrt{a^2 - b^2}}$$

for $a = 100$ and $b = 10$. By what fractional error does your estimate differ from the exact answer? (Answer: 0.01005; it differs from the true answer by only 0.004%).

3.) Taylor expanding about the point $\theta = 0$, show that

$$\sin \theta = \sum_{i=0}^{i=\infty} (-1)^i \frac{\theta^{2i+1}}{(2i + 1)!}$$

$$\cos \theta = \sum_{i=0}^{i=\infty} (-1)^i \frac{\theta^{2i}}{(2i)!}$$

4.) Two possible forms with which to express complex numbers are $a + ib$ and $re^{i\theta}$. Find the values of $r$ and $\theta$ for the following complex numbers $\alpha$: a) $\alpha = 10$; b) $\alpha = -i$; c) $\alpha = 3 + 3i$. Also, find the values of $a$ and $b$ for d) $\alpha = \exp(i\pi)$; e) $\alpha = 4 \exp(-i\pi/2)$; f) $\alpha = 2 \exp(2.0944i)$, where ‘$\exp(x)$’ means ‘$e^x$’ and $\theta$ is in radians. (Answers: 10,0; 1,3\pi/2; 4.24,\pi/4; -1,0; 0,-4; -1,1.732)

5.) Expanding the expression $\alpha = re^{i\theta}$ in terms of sines and cosines, show that $|\alpha|^2 = \alpha^*\alpha$ is real for any $r$ and $\theta$, and is equal to $r^2$. How would you demonstrate this even more easily without doing the expansion?
6.) When we introduced the Schroedinger Equation, which is complex, our oscillatory solutions went from sines and cosines to solutions of the form \( \exp(\pm ikx) \). Show that this is still an oscillatory solution by expressing the general wave function

\[
\psi = Ae^{ikx} + Be^{-ikx}
\]
in the form

\[
\psi = a \cos(kx) + b \sin(kx).
\]

What is the relation between \( a, b \) and \( A, B \)? (Answer: \( a = A + B; b = i(A - B) \).)

7.) Consider the simple harmonic oscillator solutions given in equation 6-58 in the text. 
   a) Show that \( \psi_0(x) \) satisfies the time-independent harmonic oscillator Schroedinger equation. Show that the energy of this state is \( (1/2)\hbar\omega \). 
   b) Do the same for \( \psi_1(x) \), showing that its energy is \( (3/2)\hbar\omega \). 
   c) What is the energy of a photon emitted by a transition between these two states?

8.) Problem 6.41 (Answers: a) \( k/\sqrt{2} \); b) 0.0294; c) 0.971; d) \( 9.71 \times 10^5 \))

9.) Problem 6.42 (Answers: a) \( \sqrt{3/2}k_1 \); b) 0.0102; c) 0.99; d) \( 9.9 \times 10^5 \))

10.) Problem 6.46 (Answer: \( 6.5 \times 10^{-5} \))

11.) Problem 6.47 (Answer: a) 0.111; b) 0.111)

12.) Consider a potential barrier of height \( V_0 \) which has the form of a step function at \( x=0 \).
   a) Show that, for a particle with energy \( 0 < E < V_0 \) arriving at the barrier from the left, the reflection coefficient is precisely 1 (i.e., all particles incident upon the barrier from the left will be reflected back to the right).

   b) For such a particle, the wavefunction will penetrate the barrier for some distance. In terms of \( V_0 \) and \( E \), what is the mean (expectation) value of this penetration? In other words, if you had a detector that extended to the right from \( x = 0 \), which could detect the position of particles penetrating the barrier, what would be the mean value of the position detected in this instrument?

13.) Problem 7.43 (Answers: a) 7.14 eV; b) 0.00712 eV)