Due in class Monday, 10/20/08.

Most of the answers are provided with the problems.

Reading: Tipler and Llewellyn, Sections 1.5-1.6, 2.1-2.4. Also read 2.5, but there will be no homework relating to this latter section.

1.) Problem 1.31 (Answer: 0.064)
2.) Problem 1.40. If you like, for the spacetime diagram, you can use the $\beta = 0.75$ paper handed out in class and just show that the answer is approximately correct. (Answer: a) 0.8 c; b) 4 years older than he)
3.) Problem 2.3 (Answers: a) 1.25; b) 0.383 MeV/c; c) 0.639 MeV; d) 0.128 MeV).
4.) Problem 2.8 (Answers: a) 1.51 MeV; b) 57.6 MeV; c) .346 GeV).
5.) Problem 2.10 (Answers: a) $9.0 \times 10^{13}$ J; b) $2.5$ million; c) $2.82 \times 10^{4}$ y).
6.) Problem 2.15 (Answer: a) 21 $\mu$g).
7.) Problem 2.18 (Answer: a) $4.5 \times 10^{-9}$ u; b) $7.7 \times 10^{-9}$).
8.) Problem 2.21 (Answer: 280 MeV is the Kinetic Energy of the incoming proton).
9.) Problem 2.29 (Answer: a) $\beta = 0.286$; b) 1673 MeV/$c^2$).
10.) Problem 2.45

11.) Around 1970, a great advance in particle accelerator technology was achieved by bringing two counter-circulating beams into collision, rather than directing a single beam onto a target which was fixed in the laboratory frame. Consider the annihilation of an electron and positron into a single massive particle. Calculate the mass of this particle in the case a) that an electron with energy 1 GeV collides with a positron at rest in the lab frame; b) that the 1 GeV electron collides with a positron of similar energy but opposite momentum. c) What ratio of center-of-mass energy would you expect if Newtonian, rather than relativistic, mechanics were at play here?
Answers: a) 0.032 GeV/$c^2$; b) 2 GeV/$c^2$; c) 2

12.) A pion (rest mass 140 MeV/$c^2$) decays into a muon (rest mass 105 MeV/$c^2$) and a neutrino (rest mass of 0). What is the energy of the neutrino in the rest frame of the pion? (Answer: 30.6 MeV).

13.) Consider a pion of energy 100 GeV in the laboratory frame which decays into a muon and neutrino. a) Calculate the energy of the neutrino (in the lab frame) as a function of the center-of-mass angle $\theta^*$, where $\theta^*$ is the angle of the neutrino in the pion’s rest frame
relative to the pion’s flight direction. b) What are the maximum and minimum energies of the neutrino, and for what values of $\theta^*$ does each occur? (Answer to b: nearly 0 for $\theta^* = 180^\circ$, and 43.7 GeV for $\theta^* = 0$).

14.) A new particle is observed in a particle physics experiment. You wish to measure the invariant mass of this new particle by measuring the momenta of the electron-positron pair to which this new particle happens to decay.

a) Find a formula relating the invariant mass of the new particle to the magnitude of the electron ($p_1$) and positron ($p_2$) momenta, and the ‘opening angle’ $\theta$ between their momentum vectors. Assume the opening angle is small, so that you may use the approximation

$$\cos \theta \simeq 1 - \frac{\theta^2}{2}$$

for $\theta$ in radians. Also assume that the electron and positron are energetic enough so that they are effectively massless.

b) In one observed decay of the new particle, the electron is observed to have energy 20 GeV (or momentum 20 GeV/c, if you like), the positron 40 GeV, and the angle between the electron and positron’s flight paths is observed to be .11 radians. What is the mass of the new particle?

c) What is the velocity of the new particle in this particular observation?

Answers: a) $\sqrt{p_1 p_2} \theta / c$; b) 3.1 GeV/c$^2$; c) $\beta = 0.9987$.

As a footnote, this was the technique used by one of two experiments which simultaneously discovered the 3.1 GeV/c$^2$ ‘$J/\Psi$’ particle, which discovery is now appreciated as one of the foundations of the development of the current ‘Standard Model’ of particle physics.