\[ x' = x - vt; \quad y' = y; \quad z' = z \]
\[ \beta = v/c \]
\[ \Delta t = \gamma \Delta t' \]
\[ (\Delta s)^2 = (c \Delta t)^2 - (\Delta x)^2 \]
\[ u'_x = (u_x - v)/(1 - (vu_x/c^2)) \]
\[ u'_z = u_z/\gamma[1 - (vu_x/c^2)] \]
\[ E = \gamma mc^2 \]
\[ p = (E/c, p_x, p_y, p_z) \]
\[ E^2 = (mc^2)^2 + (pc)^2 \]
\[ \nu = c/\lambda \]
\[ R = \sigma T^4; \quad \sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \]
\[ n(\lambda) = 8\pi \lambda^{-4} \]
\[ u(\lambda) = \frac{8\pi he\lambda^{-5}}{\exp(he/\lambda kT)-1} \]
\[ eV_{\text{stop}} = h\nu - \phi \]
\[ \lambda_c = 2.43 \times 10^{-12} \text{ m} \]
\[ \Delta N = \frac{l_0 A_{\text{ann}} kZ e^2}{4E_k} \frac{1}{\sin^4(\theta/2)} \]
\[ \frac{d\sigma}{d\Omega} = \frac{kZ e^2}{4E_k} \frac{1}{\sin^4(\theta/2)} \]
\[ U_E = \frac{kq_n q_s}{\lambda_{\text{NN}}} \]
\[ \frac{1}{\lambda_{\text{NN}}} = Z^2 R \left( \frac{1}{N^2} - \frac{1}{n^2} \right) \]
\[ L = n\hbar \]
\[ \lambda = h/p \]
\[ n\lambda = D \sin \theta \]
\[ p = \hbar k \]
\[ v = \omega/k \]
\[ \Delta x \Delta p_x \geq \hbar/2 \]
\[ ct' = \gamma(ct - \beta x); \quad x' = \gamma(x - \beta ct) \]
\[ \gamma = 1/\sqrt{1-\beta^2} \]
\[ L = Lp/\gamma \]
\[ \Delta \tau = \Delta s/c \]
\[ u'_y = u_y/\gamma[1 - (vu_x/c^2)] \]
\[ \tilde{p} = \gamma m\tilde{v} \]
\[ E'/c = \gamma(E/c - \beta p_x); \quad p'_x = \gamma(p_x - \beta E/c) \]
\[ M = |p|/c; \quad |p|^2 = (E/c)^2 - p_x^2 - p_y^2 - p_z^2 \]
\[ pc = \beta E \]
\[ E = h\nu \]
\[ \lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m-K} \]
\[ \overline{E} = kT \]
\[ \overline{E} = \frac{h\nu}{\exp(he/\lambda kT)-1} \]
\[ \lambda_2 - \lambda_1 = \lambda_c(1 - \cos \theta) \]
\[ b = \frac{kZ e^2}{2E_k} \cot \frac{\theta}{2} \]
\[ d\Omega = \sin \theta \, d\theta \, d\phi \]
\[ r_d = \frac{kZ e^2}{\overline{E}} \]
\[ E_n = -hcR \frac{Z^2}{n^2} \]
\[ \mu = \frac{mM}{m+M} \]
\[ \sqrt{\nu} = A_n(Z - b) \]
\[ \omega = 2\pi \nu = E/\hbar \]
\[ \lambda = h/\sqrt{2mE_k} \]
\[ E = p^2/2m \]
\[ v_y = d\omega/dk \]
\[ \Delta t \Delta E \geq \hbar/2 \]
\[ P(x) = |\psi(x, t)|^2 \]
\[ \Delta x^2 = \sum (x - \bar{x})^2 / N \]
\[ -\frac{\hbar^2}{2m} \nabla^2 \psi(x, t) + V(x) \psi(x, t) = i\hbar \frac{\partial \psi(x, t)}{\partial t} \]
\[ k = \sqrt{2m(E-V)/\hbar} \]
\[ E_n = (\hbar^2 n^2) / (8m L^2) \]
\[ \langle O \rangle = \int \psi(x)^* O \psi(x) dx \]
\[ V(x) = (1/2) m \omega^2 x^2 \]
\[ E_{n_1 n_2 n_3} = \frac{\hbar^2}{8m L^2} (n_1^2 + n_2^2 + n_3^2) \]
\[ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \]
\[ L_{op}^2 = -\hbar^2 [\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial}{\partial \theta}) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2}] \]
\[ f_{lm}(\theta) = \sin \theta \left[ \frac{e^{im\beta}}{d \cos \theta} \right] \]
\[ g(\phi) = e^{im \phi} \]
\[ I_{nl}(\rho) = (d/d\rho)^{2l+1} [e^{\rho} (d/d\rho)^{n+l} (\rho^{n+l} e^{-\rho})] \]
\[ E_1 = \frac{k^2 e^4}{2\hbar} \approx 13.6 \text{ eV} \]
Let \[ I_n = \int_0^{+\infty} x^n e^{-\lambda x^2} dx \]
\[ I_2 = \frac{1}{\sqrt{2\pi}} \quad I_3 = \frac{1}{\sqrt[3]{2\pi}} \]
\[ j_1 + j_2 \geq j_{\text{tot}} \geq |j_1 - j_2| \]
\[ PV = n RT \]
\[ \langle E \rangle = \frac{1}{2} N_{\text{dof}} kT \]
\[ f_{BE}(E) = \frac{1}{e^{(E-E_k)/kT}-1} \]
\[ n(E) = g(E) f(E) \]
\[ n(E) dE = 2\pi N \langle \frac{1}{2} \rangle \frac{1}{2} \frac{1}{2} E^{1/2} e^{-E/kT} dE \]
\[ \lambda = \frac{1}{\rho \sigma r} \]
\[ I = \frac{me^2}{\rho \sigma r m <v> L} \]
\[ c = 2.998 \times 10^8 \text{ m/s} \]
\[ \hbar = 1.054 \times 10^{-34} \text{ J-s} = 6.582 \times 10^{-16} \text{ eV-s} \]
\[ hc = 1240 \text{ eV-nm} \]
\[ e = 1.602 \times 10^{-19} \text{ C} \]
\[ m_e = 5.110 \times 10^5 \text{ eV/c}^2 = 9.109 \times 10^{-31} \text{ kg} \]
\[ R_{\infty} = 1.097 \times 10^7 \text{ m}^{-1} \]
\[ N_0 = 6.02 \times 10^{23} \]
\[ G(x; \sigma) = A \exp[1/2(x^2/\sigma^2)] \]
\[ \int P(x) dx = 1 \]
\[ \phi(t) = \exp(-i\omega t) \]
\[ -\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + V(x) \psi(x) = E \psi(x) \]
\[ K = \sqrt{2m(V-E)/\hbar} \]
\[ \psi_n(x) = \sqrt{2/L} \sin(n\pi x/L) \]
\[ p_x^{op} = \frac{\hbar}{i} \frac{\partial}{\partial x} \]
\[ \psi_{nm}(x_1, x_2) = \psi_n(x_1) \psi_m(x_2) \]
\[ -\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi = E \psi \]
\[ \nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial}{\partial r}) - L_{op}^2 / \hbar^2 \]
\[ L_{op}^2 = -i \hbar \frac{\partial}{\partial \phi} \]
\[ P_1(\theta) = (d/d\cos \theta)^l (\cos^2 \theta - 1)^l \]
\[ R(r) = r^l L_{nl}(2\pi a_0) e^{-r/(na_0)} \]
\[ E_n = -\frac{Z^2}{n^2} \]
\[ \int_0^\infty x^n e^{-x} dx = n! \]
\[ I_0 = \frac{1}{2} \sqrt{\frac{1}{\pi}} \quad I_1 = \frac{1}{2} \]
\[ I_4 = \frac{3}{8} \sqrt{\frac{1}{\pi}} \quad I_5 = \frac{1}{2} \]
\[ |J| = \sqrt{(j)(j+1)\hbar} \]
\[ C_V = \frac{1}{n} (\frac{\partial Q}{\partial T})_V \]
\[ f_D(E) = A e^{-E/kT} \]
\[ f_{FD}(E) = \frac{e^{E-E_k/kT}}{e^{E-E_k/kT}+1} \]
\[ n(v) dv = 4\pi N \langle m / 2\pi kT \rangle^{3/2} e^{-mv^2/2kT} v^2 dv \]
\[ N(x) = N_0 e^{-x/\lambda} \]
\[ V_d = \frac{E}{m \langle v \rangle} \]
\[ e = \frac{A}{L} R \]
\[ \hbar = \hbar / 2\pi \]
\[ h = 6.626 \times 10^{-34} \text{ J-s} \]
\[ k = 1.381 \times 10^{-23} \text{ J/K} = 8.617 \times 10^{-5} \text{ eV/K} \]
There are \[ 1.602 \times 10^{-19} \text{ J per eV} \]
\[ a_0 = \hbar^2 / m ke^2 = 0.0529 \text{ nm} \]
\[ k = 8.99 \times 10^9 \text{ N-m}^2 / \text{C}^2 \text{ (electrostatic constant)} \]
\[ R = N_0 k = 8.31J / 0 K \text{ - mol} \]
\[ A = 1/(\sqrt{2\pi\sigma}) \]
PROBLEM 1 [25 PTS]

Deuterium is the isotope of Hydrogen which has nucleus composed of a proton and a single neutron. The binding interaction between a proton and neutron has the rather surprising property that in the ground state, the half-integer spins of the proton and neutron are aligned, i.e., the deuterium nucleus, or ‘deuteron’, has a spin quantum number of \( s = 1 \). In the following problem, all interaction energies are small compared to the internal binding energy of the deuteron, so that we can treat the deuteron as a fundamental particle of spin 1, ignoring the fact that it is really made up of a proton and a neutron.

Consider a bound state consisting of a deuteron and antideuteron (antiproton and antineutron, also with spin quantum number \( s = 1 \)). The total angular momentum \( \vec{J} \) of this system is the sum of the two deuteron spins:

\[
\vec{J} = \vec{s}_1 + \vec{s}_2
\]

a) What values of the total angular momentum quantum number \( j \) are possible for this system?

b) What values of the total angular momentum \( |\vec{J}| \) are possible for this system?

Let \( m_j \) be the quantum number associated with the projection of the total angular momentum of the system along the \( z \) axis (the direction you ascribe to the \( z \) axis is, of course, arbitrary). For any given energy level in the bound state, assuming that there is no interaction between the deuteron and antideuteron spins, there will be a set of degenerate states corresponding to the different possible combinations of \( \vec{s}_1 \) and \( \vec{s}_2 \) into the total angular momentum \( \vec{J} \), each state given by a unique value of the quantum numbers \( j \) and \( m_j \). In other words, no matter how you combine \( \vec{s}_1 \) and \( \vec{s}_2 \) into \( \vec{J} \), you get the same energy.

c) What is the resulting degeneracy of states for each energy level of the bound system, i.e., how many unique combinations of \( j \) and \( m_j \) are possible?

d) How many of these degenerate states are associated with each possible value of \( j \) from part a)? You should have one answer for each possible value of \( j \) from part a), with the total adding up to your answer from part c).
PROBLEM 2 [25 PTS]

In the following, you may assume that the energies associated with vibrational modes are too great to be excited thermally.

Consider a gas of helium atoms contained in a box at room temperature \( T = 300 \text{K} \).

a) What is the mean kinetic energy of the He atoms in the gas?

b) What would your answer to a) be if the He atoms were restricted to move in only two dimensions?

Now consider a gas of hydrogen molecules \( (H_2) \) at room temperature. The shape of the \( H_2 \) molecule is like that of a dumbbell, with perfect symmetry about the axis of the dumbbell.

c) What is the mean kinetic energy of the \( H_2 \) molecules? (They are not restricted to move in only two dimensions.)

d) Which of the three systems described in a), b), and c) has the largest heat capacity \( C_V \)? Why?

e) Which of the three systems described in a), b), and c) has the smallest heat capacity \( C_V \)? Why?
PROBLEM 3 [25 PTS]

For a gas of distinguishable particles, we’ve seen that the relative probability of any particle having the energy \( E \) is given by the Boltzmann factor

\[
\mathbf{f}_B = e^{-E/kT}.
\]

Consider a two-dimensional ideal gas of temperature \( T \) composed of distinguishable objects of mass \( m \) whose degrees of freedom are translation in the \( x \) and \( y \) directions.

a) In terms of the velocity components \( v_x \) and \( v_y \), what is the probability of finding any given particle with \( v_x \) in the infinitesimal range between \( v_x \) and \( v_x + dv_x \), and in the infinitesimal range between \( v_y \) and \( v_y + dv_y \)? Make sure that you use the formulas for gaussian integrals from above to normalize the distribution.

b) Using the result of part a), write down an integral expression representing the probability of finding any given particle with \( v_x \) lying in the finite range between \( v_{x,1} \) and \( v_{x,2} \), for any value of \( v_y \). Your integral expression should thus not contain \( v_y \). You do not need to perform the integration.

c) For this two-dimensional ideal gas, how many particles are there with speed \( v \) between \( v \) and \( v + dv \)? In other words, what is the Maxwell distribution \( n(v)dv \) of speeds for a two-dimensional ideal gas? Assume that the gas consists of \( N \) particles.
PROBLEM 4 [25 PTS]

For the following problem, you really don’t need to remember anything about the quantum mechanical harmonic oscillator. You do need to know something about the behavior of Fermi-Dirac particles though.

As we have seen, the solutions of the Schrodinger Equation for a particle of mass \( m \) moving under the influence of a harmonic oscillator potential

\[
V(x) = \frac{1}{2}kx^2
\]

are non-degenerate states evenly spaced in energy, with energies

\[
E_n = (n - \frac{1}{2})\hbar \omega \quad n = 1, 2, 3, \ldots
\]

where \( \omega = \sqrt{k/m} \). Consider a system of \( N \) identical fermions in motion inside of such a potential well. Feel free, if you like, to ignore the \( 1/2\hbar \omega \) term in the expression for the energy, i.e., to set \( E_n = n\hbar \omega \). (Don’t neglect the spin of the electron).

a) What is the Fermi energy \( E_F \) for this system, i.e., the energy of the highest-energy occupied state of the system at \( T = 0 \) (absolute 0). Express your answer in terms of \( N \) and \( \omega \).

b) Write down an expression showing the number density function \( N(E) \) for this system, for any temperature \( T \). You may express your result in terms of \( E_F \), rather than substituting in your result from a).

c) What is the average energy per particle at \( T = 0 ? \) Again, you may express your result in terms of \( E_F \) if you like.

d) As in homework 4, approximate the effect of temperature on the distribution function as moving all particles with \( E_F - kT < E < E_F \) (within \( kT \) below the Fermi energy) to the range \( E_F < E < E_F + kT \) (within \( kT \) above the Fermi energy). Find an expression for the total energy in the system as a function of temperature. Again, you may express your answer in terms of \( E_F \) if you like.

e) Use this to estimate the total heat capacity at constant volume of the system (you need not convert it to the heat capacity per mole, which is the formal definition of \( C_V \)). Does this result depend upon the temperature?