Concave mirror, with radius of curvature 60 cm. It asks for \( f = \frac{R}{2} = 30 \text{ cm} \).

What's \( s' \)?

\[
\frac{1}{f} = \frac{1}{s'} + \frac{1}{s} \Rightarrow s' = 90 \text{ cm}
\]

Object distance = 45 cm

Find magnification.

\[
m = -\frac{s'}{s} = -\frac{90 \text{ cm}}{45 \text{ cm}} = -2
\]

Real and inverted.

Now it asks you to solve the mirror equation for \( s' \):

\[
\frac{1}{f} = \frac{1}{s'} + \frac{1}{s} \\
\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} \\
s' = \frac{1}{\frac{1}{f} - \frac{1}{s}}
\]

\[
s' = \frac{fs}{s-f}
\]

\[
m = -\frac{s'}{s} = -\frac{f}{s-f}
\]
... what's the image like for a similar mirror, but convex? (f = -30 cm)

Based on equations we just derived, 
\[ s' = \frac{fs}{s-f} \], so if f is negative, s' is negative, thus real. 
\[ m' = \frac{-f}{s-f} \] will be positive. Or:

\[ \text{magnification is evidently less than one.} \]

For a mirror with infinitely large radius of curvature (flat mirror), 
\[ \frac{1}{f} = \frac{1}{s'} + \frac{1}{s} \] yields 
\[ s' = -s \]
Ray Tracing and Image Formation with Spherical Mirrors:

A) A convex mirror always forms an upright, reduced image:

B) If the radius of curvature is 50 cm, how far away \( d_o \) should the object be placed to get an image 20 cm away?

\[
\frac{1}{f} = \frac{1}{5} + \frac{1}{5'}
\]

\[
\frac{1}{25} = \frac{1}{d_o} + \frac{1}{20} \quad \Rightarrow \quad d_o = -100 \text{ cm}
\]

c) Allowed ray tracings: (concave mirror)

D) Now for concave mirror, if the object is still 100 cm away, where will the image be?

\[
\frac{1}{f} = \frac{1}{5} + \frac{1}{5'}
\]

\[
\frac{1}{25} = \frac{1}{100} + \frac{1}{5'} \quad \Rightarrow \quad s' = 33.3 \text{ cm}
\]
Spherical Mirror 1

You want to create an image 10 m from an object, inverted and half the height of object, using one spherical mirror.

We know \( m = -\frac{1}{2} = -\frac{s'}{s} \) so \( s = 2s' \)

Also, \( s - s' = 10 \), so

\[ 2s' - s' = 10 \]

\[ s' = 10 \text{, thus } s = 20 \]

\[ f = \frac{1}{\frac{s}{s' + s}} = \frac{s}{1 + \frac{s}{s'}} = \frac{ss'}{s + s'} = \frac{10 \cdot 20}{10 + 20} = \frac{200}{30} = 6.67 \text{ m} \]

- must be concave, since \( f \) is positive

- Radius of curvature must be \( 2 \cdot f = 13.3 \text{ m} \)

- Real image

\[ \text{object} \]

[Diagram of spherical mirror showing object, image, and the ray paths]
Spherical Mirror 2

You want to create an image 10 m from an object, upright, half the size, using one spherical mirror.

What's $F$?

$m = \frac{1}{2} = -\frac{s'}{s}$ so $s = -2s'$

$s - s = 10$
$s' - (-2s') = 10$
$3s' = 10$
$s' = 10/3$ so $s = -20/3$

$f = \frac{ss'}{s+s'} = -6.67 \text{ m}$

Must be convex, since $f$ is negative.

Radius = $2F = 13.3 \text{ m}$

Virtual, since negative image distance (opposite sign of $s$, anyways)

![Diagram of a spherical mirror with object, image, and focal length labeled.]
Image Size in a Mirror

**Joe**

\[ f = -15 \text{, since } R \text{ is convex.} \]

How far away does his image appear?

\[ \frac{1}{f} = \frac{1}{s'} + \frac{1}{s} \]

\[ -15 = \frac{1}{s'} + \frac{1}{5} \rightarrow s' = -3.75 \text{m} \]

Add -5 m for answer: 8.75 m

What's his image's height?

\[ m = -\frac{s'}{s} = -\frac{-3.75}{5} = 0.75 \text{, multiply his actual height by this factor to get } 1.2 \text{ m} \]

Now he falls on his face, with feet fixed.

What's his image's length?

First calculate his head's image position:

\[ \frac{1}{15} = \frac{1}{3.4} + \frac{1}{s'} \quad s' = -2.772 \text{ m, difference between this value and his feet's image } (-3.75 \text{ m}) \text{ is the answer: } 0.9783 \text{ m} \]
Ray Tracing and Image Formation with a concave Lens:

If the focal length is -7.5 cm, where should the object be placed so that its image is 3.7 cm from lens?

\[-\frac{1}{7.5\text{ cm}} = \frac{1}{s} + \frac{1}{-3.7} \Rightarrow s = 7.3 \text{ cm}\]

For concave lens, s, s' on same side (negative)

What's m? \[m = -\frac{s'}{s} = .507\]

To increase m, move object closer to lens.

The focal length of a lens:

An object is located 28 cm from a lens. The image is real and twice as high. What's f?

\[\frac{1}{f} = \frac{1}{s'} + \frac{1}{s} \quad f = \frac{ss'}{s + s'} = \frac{(28)(56)}{28 + 56} = 18.7 \text{ cm}\]

B) Now if everything's the same, but concave lens, object 5 m high, how high is the image?

\[-\frac{1}{18.7} = \frac{1}{s} + \frac{1}{-28} \quad s' = -11.2 \quad m = -\frac{s'}{s} = .4 \quad .4 \times 5\text{ cm} = 2\text{ cm}\]
Refraction at a Spherical Surface

Object image relation: (for refraction at a spherical surface)

\[ \frac{N_a}{S_a} + \frac{N_b}{S_b} = \frac{N_b-N_a}{R} \]

Object dist. \rightarrow image dist. \rightarrow refractive indices of corresponding material \( (n_a \text{ on left}) \)

radius of curvature \( (5 \text{ cm}) \)

What's \( S_b \), the image position?

\[ \frac{N_a}{0} + \frac{1.6}{S_b} = \frac{N_b-n_a}{R} \]

\[ S_b = 13.3 \text{ cm} \] (on right, since positive)

What about if the object were, instead, placed 15 cm from left end?

\[ \frac{1}{-15 \text{ cm}} + \frac{1.6}{S_b} = \frac{1.6-1}{5 \text{ cm}} \]

\[ S_b = 30 \text{ cm} \] (on right, since positive)

Again, for \( S_a = -3 \text{ cm} \) from left end:

\( S_b \) yields \(-7.5 \text{ cm} \)... (on left side, virtual, since negative)
Chromatic Aberration

Lenses typically have different focal lengths for different wavelengths. Consider one with \( f_{\text{red}} = 19.57 \text{ cm} \), \( f_{\text{blue}} = 18.87 \text{ cm} \).

If an object 5 cm tall is 30 cm away, find the ratio: \[
\frac{\text{height of red image}}{\text{height of blue image}} \rightarrow \text{to get this...}
\]

\[
\frac{1}{s} = \frac{1}{S} + \frac{1}{S'} \quad \Rightarrow \quad m = \frac{S}{S'} \quad \text{height of red} = m \times 5
\]

follow same procedure as for red image...

answer: 1.11

B) What would you expect to see if a circular piece of white paper with radius 5 cm were placed 30 cm from the lens, centered on axis?

\[
\rightarrow \text{it would have red edges.}
\]
"One lens' image is another lens' object"
- Confucius

In this system, we have the following:

plane mirror @ x = 0

converging lens with focal length 5 m @ x = 12.5 m

First, find the location of the image from lens:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \rightarrow \frac{1}{5\text{m}} = \frac{1}{10\text{m}} + \frac{1}{s'}$$

$$s' = 10\text{m}, \text{ so the image location is } 12.5\text{m} - 10\text{m} = 2.5\text{m}$$

Second, find the image after it reflects on plane:

$$\frac{1}{\infty} = \frac{1}{s} + \frac{1}{s'} \rightarrow \frac{1}{s'} = -s = -2.5\text{m}$$

Next, this reflects through the lens again. Find $s'$:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \rightarrow \frac{1}{5\text{m}} = \frac{1}{(2.5+12.5)} = \frac{1}{s'}$$

$$s' = 7.5\text{m}, \text{ so final location is } 12.5\text{m} + 7.5\text{m} = 20\text{m}$$

real final image
(since the light actually reaches the location of the image)

Now, find $|m|$, after passing through just the 1st lens:

$$|m| = \left| \frac{-s'}{s} \right| = \frac{10\text{m}}{10\text{m}} = 1$$
No magnification occurs for plane mirror either, but for last pass through lens:

\[ |m'| = \left| \frac{-5}{-5} \right| = \frac{2.5}{15} = 0.5 \]

Total magnification = 1 * 1 * 0.5 = 0.5
upright image

"A Two - Lens System"

Object 1 cm tall

x = -50 cm

f = +10 cm

x = -20 cm

f = +8 cm

Final image location?

After 1st lens: \( \frac{1}{10} = \frac{1}{s'} + \frac{1}{30} \) \( \Rightarrow s' = 15 \) cm, so position is \( -20 + 15 = -5 \) cm

After 2nd lens: \( \frac{1}{8} = \frac{1}{s''} + \frac{1}{25} \) \( \Rightarrow s'' = 11.8 \) cm, so position is \( 20 + 11.8 = 31.8 \) cm

Magnification = \( m_1 \times m_2 = \left( \frac{-5}{5} \right) \left( \frac{-5}{-5} \right) = \left( \frac{-15}{30} \right) \left( \frac{-11.8}{25} \right) = 0.236 \)

Final height = \( (1 \text{ cm})(0.236) = 1 \text{ cm} \times 0.236 = 0.236 \text{ cm} \)

To achieve the same effect with 1 mirror at \( x = 0 \)

\( \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \) \( \Rightarrow \frac{1}{f} = \frac{1}{31.8} + \frac{1}{50} \) \( \Rightarrow f = 19.4 \text{ cm} \)
A microscope for Biology

\[
\text{adjustable length } d \quad \xrightarrow{\text{sample, 1.3 cm from objective lens}}
\]

\[
\text{lens } f = 2.5\, \text{cm} \quad \text{objective lens } f = 1\, \text{cm}
\]

What length should you choose so that the sample is in focus with completely relaxed eye? (Image must be at \(\infty\) for this to occur.)

After passing through objective:

\[
\frac{1}{F} = \frac{1}{s} + \frac{1}{s'} \quad \Rightarrow \quad \frac{1}{1\, \text{cm}} = \frac{1}{1.3} + \frac{1}{s'} \quad \Rightarrow \quad s' = 4.33\, \text{cm} \quad \text{to left}
\]

After passing through second lens:

\[
\frac{1}{2.5\, \text{cm}} = \frac{1}{d - 4.33\, \text{cm}} + \frac{1}{\infty} \quad \Rightarrow \quad d = \frac{2.5\, \text{cm}}{1 - \frac{1}{4.33\, \text{cm}}} = 6.83\, \text{cm}
\]