Center for Energy Bin for Power Law Spectrum

Aous Abdo

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Following the footsteps of Bob Ellsowrth in his short memo, I find the follwoing:

For a power law spectrum with spectral index α, the differential flux is given by:

$$\frac{dN}{dE} = AE^{-\alpha} \tag{1}$$

The number of events with energy greater than or equal to a given energy E is:

$$N(\geq E) = \frac{A}{\alpha - 1} E^{-(\alpha-1)} \tag{2}$$

The bin center should be equal to the value of energy at which the number of events greater than or equal to that energy, E_c, minus the number of events greater than or equal to the low bin edge energy, E_1, is equal to the number of events greater than or equal to the high bin edge energy, E_2, minus the number of events greater than or equal to that energy, E_c i.e.:

$$N(\geq E_c) - N(\geq E_1) = N(\geq E_2) - N(\geq E_c) \tag{3}$$

which simplifies to:

$$2 \times N(\geq E_c) = N(\geq E_2) + N(\geq E_1) \tag{4}$$

$$N(\geq E_c) = \frac{1}{2} \left[N(\geq E_2) + N(\geq E_1) \right] \tag{5}$$

Now plugging in the corresponding numbers form equation 2, we have:

$$
\begin{aligned}
N(\geq E_c) &= \frac{A}{2(\alpha-1)}\left[E_2^{-(\alpha-1)} + E_1^{-(\alpha-1)}\right] && (6)\\
\frac{A}{(\alpha-1)}E_c^{-(\alpha-1)} &= \frac{A}{2(\alpha-1)}\left[E_2^{-(\alpha-1)} + E_1^{-(\alpha-1)}\right] && (7)\\
E_c^{-(\alpha-1)} &= \frac{1}{2}\left[E_2^{-(\alpha-1)} + E_1^{-(\alpha-1)}\right] && (8)\\
E_c &= \left(\frac{1}{2}\left[E_2^{-(\alpha-1)} + E_1^{-(\alpha-1)}\right]\right)^{\frac{-1}{(\alpha-1)}} && (9)
\end{aligned}
$$

In Bob's memo we can extract E_c from the last equation:

$$
\begin{aligned}
E_c^{-\alpha} &= \frac{E_1^{-(\alpha-1)} - E_2^{-(\alpha-1)}}{(\alpha-1)(E_2-E_1)} && (10)\\
E_c &= \left(\frac{E_1^{-(\alpha-1)} - E_2^{-(\alpha-1)}}{(\alpha-1)(E_2-E_1)}\right)^{(\frac{-1}{\alpha})} && (11)
\end{aligned}
$$

But now how do we know which one is right. well for a flat spectrum, i.e. $\alpha = 0$, E_c should be equal to $\frac{(E_2+E_1)}{2}$.
Bob's equation for E_c gives:

$$
\begin{aligned}
E_c &= \left(\frac{E_1-E_2}{-1\times(E_2-E_1)}\right)^{-\infty} && (12)\\
E_c &= \left(\frac{E_1-E_2}{E_1-E_2}\right)^{-\infty} && (13)\\
E_c &= 1. && (14)
\end{aligned}
$$

While equation 9 gives:

$$
\begin{aligned}
E_c &= \left(\frac{1}{2}\left[E_2+E_1\right]\right)^{\frac{-1}{(0-1)}} && (15)\\
E_c &= \frac{E_2+E_1}{2} && (16)
\end{aligned}
$$

which is the correct answer. So I conclude that equation 9 is the correct formula for calculating the center for an energy bin for a power law spectrum.