Energy Weighting with High Energy Cutoff

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In this memo I calculate the energy weight factor ω for MC events with a high energy cut-off and which were simulated with a spectral index that is different from that of the originally thrown MonteCarlo.

In section 2.4 of [1], Robert Ellsworth derived the energy weighting that each event should be weighted with if this event were generated with a spectral index that is different from the one that the MonteCarlo simulations were thrown with assuming that there is no energy cutoff. If an energy cutoff in the spectrum is presented then the weighting will be different.

I assume the following:

The MC events were generated with spectral index α_1.

The same number of events are generated with spectral index α_2 with a high energy cutoff of E_{cut}.

The energy distribution of the generated events is

$$\frac{dN_1}{dE} = a_1 E^{-\alpha_1} \tag{1}$$

and the total number of events is

$$N_1 = \left(\frac{a_1}{\alpha_1 - 1}\right) E_{min}^{(1-\alpha_1)} \tag{2}$$

For the events that were generated with a high energy cutoff the total number of events is equal to

$$N_2 = \left(\frac{a_2}{1 - \alpha_2}\right) \left[E_{cut}^{(1-\alpha_2)} - E_{min}^{(1-\alpha_2)}\right] \tag{3}$$

The Number of events with energy cutoff N_2 is different from that for no energy cutoff N_1. To assign a weight for an event in the second sample we divide the no-cutoff sample into two parts, one part extends from E_{min} to E_{cut} and the second one from E_{cut} to ∞. Any event with energy in the range $E_{cut} < E < \infty$ is assigned a weight of zero, the weight for an event with energy in the range $E_{min} < E < E_{cut}$ is calculated as follows:

The number of events in the first sample with spectral index α_1 up to the cutoff energy is:

$$N_1' = N_1(E_{min} < E < E_{cut}) = \left(\frac{a_1}{1-\alpha_1}\right)\left[E_{cut}^{(1-\alpha_1)} - E_{min}^{(1-\alpha_1)}\right] \quad (4)$$

Now for the energy range $E_{min} < E < E_{cut}$ N_2 and N_1 are equal:

$$N_2 = N_1(E_{min} < E < E_{cut}) \quad (5)$$

$$\left(\frac{a_2}{1-\alpha_2}\right)\left[E_{cut}^{(1-\alpha_2)} - E_{min}^{(1-\alpha_2)}\right] = \left(\frac{a_1}{1-\alpha_1}\right)\left[E_{cut}^{(1-\alpha_1)} - E_{min}^{(1-\alpha_1)}\right] \quad (6)$$

and we have

$$\frac{a_2}{a_1} = \left(\frac{\alpha_2 - 1}{\alpha_1 - 1}\right)\left[\frac{E_{cut}^{(1-\alpha_1)} - E_{min}^{(1-\alpha_1)}}{E_{cut}^{(1-\alpha_2)} - E_{min}^{(1-\alpha_2)}}\right] \quad (7)$$

The ratio of the triggered events to thrown events should be the same for different spectral indices and should be only a function of the energy:

$$\frac{dN_1^{trig}/dE}{dN_1^{thrown}/dE} = \frac{dN_2^{trig}/dE}{dN_2^{thrown}/dE} \quad (8)$$

and

$$\frac{dN_2^{trig}}{dE} = \frac{dN_2^{thrown}}{dE} \times \frac{dN_1^{trig}/dE}{dN_1^{thrown}/dE} \quad (9)$$

$$\frac{dN_2^{trig}}{dE} = \frac{dN_1^{trig}}{dE} \times \frac{dN_2^{thrown}/dE}{dN_1^{thrown}/dE} \quad (10)$$

$$\frac{dN_2^{trig}}{dE} = \frac{dN_1^{trig}}{dE} \times \frac{a_2 E^{-\alpha_2}}{a_1 E^{-\alpha_1}} \quad (11)$$

$$\frac{dN_2^{trig}}{dE} = \frac{dN_1^{trig}}{dE} \times \left(\frac{\alpha_2 - 1}{\alpha_1 - 1}\right) E^{-(\alpha_2-\alpha_1)} \left[\frac{E_{cut}^{(1-\alpha_1)} - E_{min}^{(1-\alpha_1)}}{E_{cut}^{(1-\alpha_2)} - E_{min}^{(1-\alpha_2)}}\right] \quad (12)$$

so every triggered event with energy E shoud be weighted by:

$$\omega(E_{min} < E < E_{cut}) = \left(\frac{\alpha_2 - 1}{\alpha_1 - 1}\right)\left(\frac{E}{E_{min}}\right)^{(\alpha_1-\alpha_2)} \Gamma \quad (13)$$

$$\omega(E_{cut} < E < \infty) = 0. \quad (14)$$

where $\Gamma = \Gamma(E_{cut}, E_{min}, \alpha_1, \alpha_2) = \left[\frac{(E_{cut}/E_{min})^{(1-\alpha_1)}-1}{(E_{cut}/E_{min})^{(1-\alpha_2)}-1}\right]$

for $E_{cut} \to \infty$ (no cutoff) the above expression reduces to

$$\omega = \left(\frac{\alpha_2 - 1}{\alpha_1 - 1}\right)\left(\frac{E}{E_{min}}\right)^{(\alpha_1-\alpha_2)} \quad (15)$$

which is the expression given in Bob's memo. Furthermore, since Γ is independent of E, the energy dependence of the weights in 13 and 15 is identical.

Figure 1 shows the Γ factor as a function of the cut-off energy for simulated MC events. For this plot $\alpha_1 = 2.75$, $\alpha_2 = 2.65$, and $E_{min} = 30$ GeV.

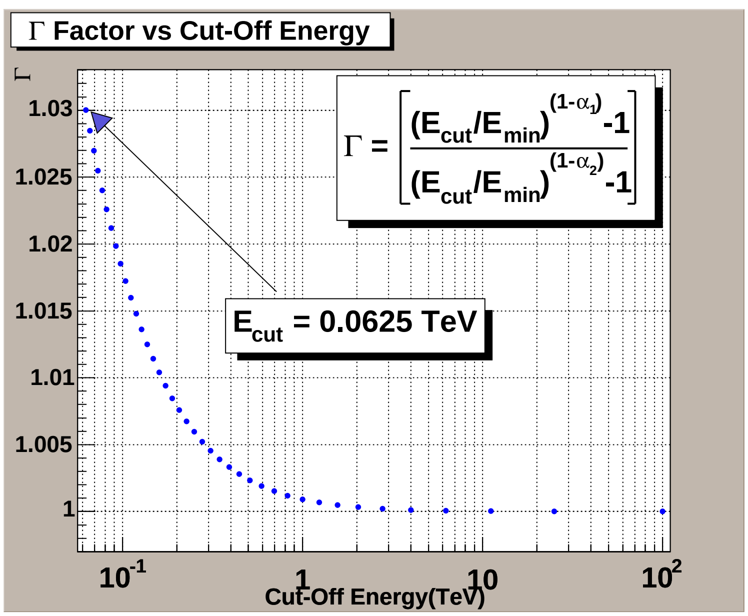

Figure 1: Gamma Factor as a function of the cut-off energy. For this plot $\alpha_1 = 2.75$, $\alpha_2 = 2.65$, and $E_{min} = 30$ GeV.

From Figure 1 we see that the Γ factor has a maximum value of 1.03 at $E_{cut} = 62.5$ GeV. We also notice that Γ approaches the value of 1.0 very quickly as E_{cut} increases. Thus the effect of the cutoff on weighting turns out to be very small.

Figure 2 shows the weight factor ω as a function of energy for simulated MC events. For this plot $\alpha_1 = 2.75$, $\alpha_2 = 2.65$, $E_{min} = 30$, and $E_{cut} = 100$ TeV.

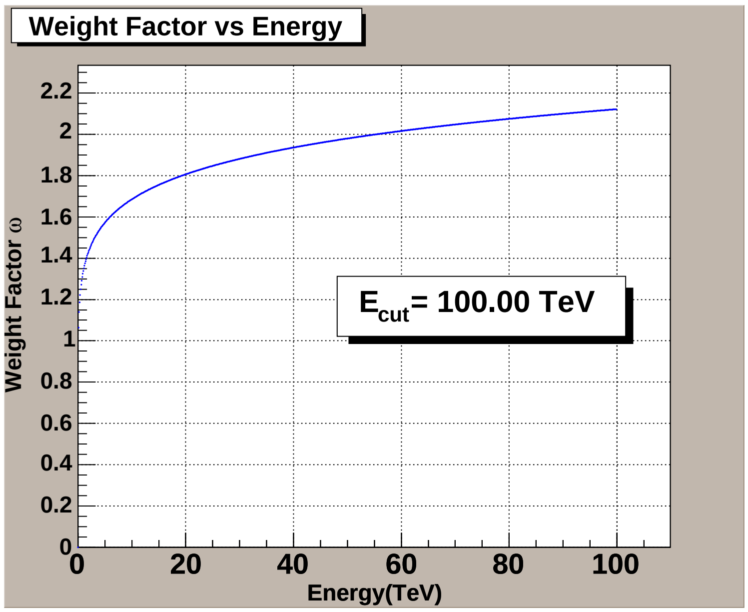

Figure 2: Weight Factor ω as a function of energy. For this plot $\alpha_1 = 2.75$, $\alpha_2 = 2.65$, $E_{min} = 30$ GeV, and $E_{cut} = 100$ TeV.

References

[1] Robert Ellsworth, "Calculation of the Trigger Rate from Geant4 Monte Carlo Data"
Milagro Internal Memo.