Calculation of the Trigger Rate from Geant 4 Monte Carlo Data

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1 Introduction

This note describes a method to calculate the trigger rate from Monte Carlo results and data on primary fluxes.

To increase the efficiency of the Monte Carlo calculation, primary particles are thrown with a non-uniform radial distribution. To compensate for this, distributions of Monte Carlo quantities are all weighted by the core radius. The way this is included in the trigger rate calculation is described here.

2 Rate Calculation with no MC radial weighting

Consider a detector of area A [1], with perfect detection efficiency, sitting in an isotropic flux (with no upward momenta). Let θ be the zenith zenith angle, and E the primary energy. For this detector (at the top of the atmosphere), the event rate is

$$\frac{dN}{dt} = \int\int\int \frac{d^4N}{dEd\Omega dAdt} \cos\theta \; dA \; d\Omega \; dE$$

The first term in the integral is the differential primary flux. Primary particles strike the area A uniformly. The factor $\cos\theta$ accounts for the fact that the area is effectively smaller for larger zenith angles. The Monte Carlo calculation includes this factor in generating primary particles.

Using

$$d\Omega = -2\pi d(\cos\theta)$$

the first equation becomes

$$\frac{dN}{dt} = -2\pi \int\int\int \frac{d^4N}{dEd\Omega dAdt} \; \cos\theta \; dA \; d\cos\theta \; dE$$

The number of incident particles per unit time is then the number incident on area of the circle with the maximum throw radius,R_{throw}. That is, in the unweighted Monte Carlo

$$\frac{dN_{throw}}{dt} = 2\pi(\pi R_{throw}^2) I(> E_{min})(1 - \cos{\theta_{max}}^2)/2$$

Here $E_{min}$and θ_{max} are the minimum energy and maximum angle of primary particles.

The number of triggers at detector level, per unit time is

$$\frac{dN_{trig}}{dt} = -2\pi \int\int\int \frac{d^4N}{dEd\Omega dAdt} \cos\theta \; \epsilon(E,\theta,r) \; dA \; d\cos\theta \; dE$$

[1] For the Milagro detector, this area is the area of a circle with radius equal to the maximum Monte Carlo throw radius, R_{throw}

where ϵ is an efficiency which depends on E, θ, radius r, and the various cuts.

But ϵ does not have to be determined. Instead, note that

$$\frac{dN_{trig}}{dt} == \frac{dN_{throw}}{dt}\frac{N_{trig}}{N_{throw}} \equiv \frac{dN_{throw}}{dt}\eta \tag{1}$$

where

$$\eta = \frac{N_{trig}}{N_{throw}} \tag{2}$$

This can be determined from the Monte Carlo results.

3 Calculation with weighting in Monte Carlo

3.1 Radial Weighting

The equation above holds for a Monte Carlo simulation in which the cores are thrown uniformly on the array. To increase the efficiency of the of the simulation, the core radii are thrown with a non-uniform distribution:

$$\frac{dn}{dr} = b$$

in which b is a constant, and $0 < r < R_{throw}$. The Monte Carlo data distributions are corrected by weighting every event in any histogram by its core radius r.

With these procedures it is necessary to find the correct way to write Equations 1 and 2.

3.2 Case with no r-weighting, again

Here

$$\frac{dN_{throw}}{dr} = ar$$

and, by integration,

$$N_{throw} = aR_{throw}^2/2$$

$$\frac{dN_{trig}}{dr} = \frac{dN_{throw}}{dt}\beta(r) = ar\beta(r)$$

Here $\beta(r)$ is an efficiency with only r-dependence. The number of triggers is then

$$N_{trig} = a\int r\beta(r)dr$$

and the overall efficiency η is

$$\eta = \frac{N_{trig}}{N_{throw}} = \frac{2\int r\beta(r)dr}{R_{throw}^2} \tag{3}$$

Note that the factors a have cancelled.

3.3 Case with r-weighting

With r-weighting the same quantities as in the previous subsection are calculated:

$$\frac{dn_{throw}}{dr} = b$$

$$N_{throw} = bR_{throw}$$

and

$$\frac{dN_{trig}^{wtd}}{dr} = \frac{dN_{throw}}{dt} r\beta(r) = br\beta(r)$$

Integrating,

$$N_{trig}^{wtd} = b \int r\beta(r)dr$$

So the efficency is

$$\frac{N_{trig}^{wtd}}{N_{throw}} = \frac{\int r\beta(r)dr}{R_{throw}}$$

In order to have η the same in the weighted and unweighted cases, it is necessary to weight the number generated by a factor $R_{throw}/2$. Then

$$\eta = \frac{N_{trig}^{wtd}}{N_{throw}} \frac{2}{R_{throw}} = \frac{2\int r\beta(r)dr}{R_{throw}^2}$$

which is the same as Equation 3

For the current MC runs, $\theta_{max} = 70°$, so the trigger rate is

$$\frac{dN_{trig}}{dt} = \pi(\pi R_{throw}^2) I(> E_{min})(1.766)(\frac{N_{MC,trig,wtd}}{N_{MC,throw} R_{throw}})$$

3.4 Correction when MC index $\neq$ true index

The MC helium data is currently generated with spectral index 2.75. Fits to the BessTeV data give an index of 2.65. So a correction must be made to find the number of triggers which would have been found if the spectral index had been 2.65.

Suppose the MC events were generated with index γ_1. Then the energy distribution of the generated events is

$$\frac{dN_1}{dE} = b_1 E^{-\gamma_1}$$

and the total number of events is

$$N_1 = b_1 E_{min}^{-(\gamma_1 - 1)}/(\gamma_1 - 1)$$

Now suppose the **same number** of events were generated with index γ_2. Then because the number of events is the same in each run,

$$b_1 E_{min}^{-(\gamma_1-1)}/(\gamma_1 - 1) = b_2 E_{min}^{-(\gamma_2-1)}/(\gamma_2 - 1)$$

So,

$$b_2/b_1 = \frac{(\gamma_2 - 1)}{(\gamma_1 - 1)} E_{min}^{\gamma_2 - \gamma_1}$$

Now suppose the energy distribution of triggers with the wrong γ (γ_1) is

$$\frac{dN_{trig,1}}{dE} = \frac{dN_{throw,1}}{dE}\alpha(E)$$

where $\alpha(E)$ is an efficiency which does not depend on γ, then

$$\frac{dN_{trig,2}}{dE} = \frac{dN_{throw,2}}{dE}\alpha(E)$$

or

$$\frac{dN_{trig,2}}{dE} = \frac{dN_{throw,2}}{dE}\frac{dN_{trig,1}}{dE}/\frac{dN_{throw,1}}{dE}$$
$$= \frac{dN_{trig,1}}{dE}\frac{b_2 E^{-\gamma_2}}{b_1 E^{-\gamma_1}}$$
$$= \frac{dN_{trig,1}}{dE}\frac{(\gamma_2 - 1)}{(\gamma_1 - 1)}(\frac{E}{E_{min}})^{-\gamma_2+\gamma_1}$$

So every triggered event with energy E should be multiplied by a weight factor

$$w = \frac{(\gamma_2 - 1)}{(\gamma_1 - 1)}(\frac{E}{E_{min}})^{\gamma_1 - \gamma_2}$$

A plot of the weight factor vs. energy, for $\gamma_1 = 2.75$ and $\gamma_2 = 2.65$ is shown in Figure 1

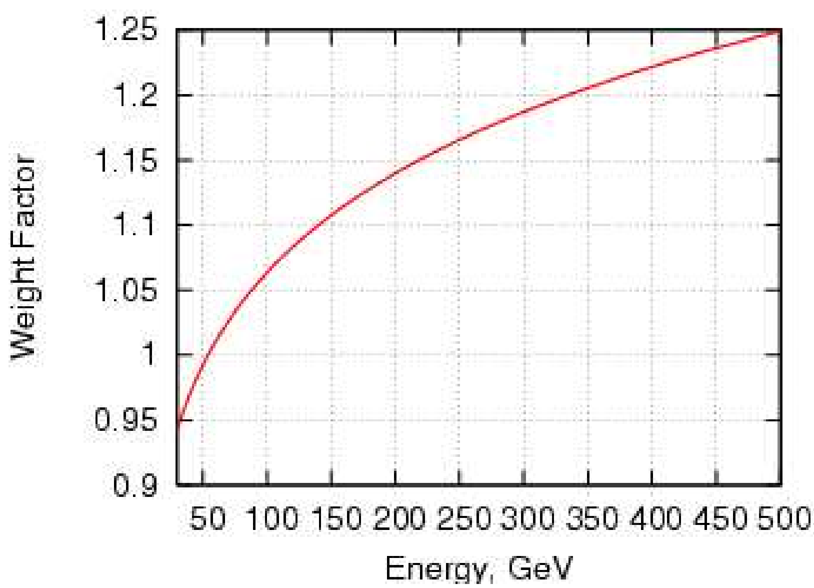

Figure 1: Weight factor for $\gamma_1 = 2.75$, $\gamma_2 = 2.6$, and $E_{min} = 30\ GeV$

4 Current Results

The Milinda code was run on some of the latest Geant4 MC (Version 1.2) events. The number of r-weighted triggered events was found. To better simulate the running experiment, 37 tubes were made "dead". The number of generated events was determined with Vlasios' database. The trigger for this study is the VME trigger.

4.1 Fluxes

For a differential flux

$$\frac{d^4N}{dE\ dA\ d\Omega\ dt} = BE^{-\gamma}$$

the corresponding integral flux is

$$I(> E_{min}) = BE_{min}^{-(\gamma-1)}/(\gamma - 1)$$

For this calculation, $B_{proton} = 15481.$, $\gamma_{proton} = 2.75$ $B_{helium} = 6369$, $\gamma_{helium} = 2.65$

4.2 Proton trigger rates

For Geant 4, VME trigger rate = 1511 $/sec$.

4.3 Helium trigger rates

For Geant 4, VME trigger rate = 517 $/sec$

4.4 Total

So the total VME predicted trigger rate is 2028/s. Helium triggers are about 26% of the total. These rates should still be regarded as preliminary.