WIMP Annihilation in the Solar Neighborhood: a Study of Optimal Bin Size and Comments on Background Methods

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For a very long time, many different experiments have indirectly searched for evidence of Weakly Interacting Massive Particles (WIMPs, here denoted χ) by looking for evidence of their annihilation products. Perhaps the most common example is the search for neutrinos from WIMP annihilation in the Sun and in the Earth. Needless to say that no such evidence has yet been found in part perhaps because of the difficulty in detecting what turns out to be a relatively weak neutrino flux. Many people have discussed the possibility of indirect detection by seeing the annihilation line (or annihilation spectrum) due to e.g. $\chi\chi\rightarrow\gamma\gamma$ or similar final states. Steven Strausz has suggested detection of this signal by looking in the neighborhood of the Sun for the annihilation gamma-rays. In this note, I explore the appearance of the signal around the Sun so that we can begin to develop an idea of how to search for the signal.

Calculation of the distribution of WIMPs in the Solar System is a difficult proposition. The precise distribution will depend upon some known and some unknown quantities. I will not explore all of the possibilities and parameters that are relevant for this problem in this note as Roman, Lazar, S. Habib, G. Jungmann and I are doing a numerical study. Instead, I want to get an idea of how the signal around the Sun might look, assuming there is a detectable flux of gamma-rays in the solar neighborhood. With this information, we can begin to formulate methods of attacking this difficult analysis problem.

Without a detailed understanding of the distribution of WIMPs in the Solar System, I will assume the distribution of annihilations around the Sun can be parameterized naively as a power-law

$$P(r) = a\left(\frac{r}{R_*}\right)^{-b}$$

where R_* is the solar radius and b is some parameter that describes how the annihilation rate decreases with increasing distance from the Sun; the parameter a has units of e.g. s^{-1} cm^{-3}. Clearly I am ignoring any accumulation of WIMPs in the Earth's gravitational potential, any perturbations caused by planets, and am assuming the annihilation is spherically symmetric around the Sun and decreasing as distance away from the Sun.

The WIMP signal, if present, must be extracted from the region around the Sun. Complicating this analysis is the presence of the distorted, shifted cosmic-ray Sun shadow. The shape of this shadow will have to be understood in some detail. Less well known is the gamma-ray signal from the Sun due to π^0's produced in helioatmospheric interactions of cosmic rays; the gamma-rays are produced near the Sun's limb. Seckel, Stanev and Gaisser [Astroph. J. 382, 652(1991)] have studied this in some detail and predict an integral flux of ~10^{-8} – 10^{-9} cm^{-2} s^{-1} above 10 GeV, with a cosmic-ray like spectral index; this signal should be detectable by Milago, and is worthy of study in its own right. It will also provide important information to aid out understanding of the solar shadow. Moreover, the WIMP signal has a distinctive energy spectrum that should make it separable from these effects. The solar shadow and helioatmospheric gamma-ray effects are left for later study. In this note, I am interested in understanding the appearance of the WIMP signal in the region around the Sun. I will only give a single example of the effects of Milagro's angular resolution.

Analytic Calculations & Numerical Results

We are interested in the total gamma-ray flux at Earth from WIMP annihilation. Given l as the line-of-sight distance from Earth, we can write (in spherical coordinates with the Earth-Sun line as the z-axis)

$$dN\gamma = \frac{aAR_*^b l^2 d(l) d(\cos\vartheta) d\varphi}{4\pi l^2 r^b}$$

$$or$$

$$\frac{dN\gamma}{d(\cos\vartheta)} = \frac{aAR_*^b}{2}\frac{d(l)}{r^b} \qquad eq.1$$

where A is an area element on Earth, and the l^2 in the numerator (denominator) is from the spherical volume element (solid angle of area A at a distance l), and again r is the distance from (l,θ,φ) to the Sun's center. This needs to be integrated from $l = 0$ (at Earth) to infinity unless the direction is pointing at the Sun's disk, in which the upper limit to the integration is L_* (the distance to the Sun, 1 AU). Using the law-of-cosines,

$$r^2 = L_*^2 + l^2 - 2L_* l \cos\vartheta$$

which can be substituted into the above expression eq.1

$$\frac{dN\gamma}{d(\cos\vartheta)} = \int_0^{l_{max}} \frac{aAR_*^b}{2}\frac{d(l)}{\left[L_*^2 + l^2 - 2L_* l\cos\vartheta\right]^{\frac{b}{2}}} \qquad eq.2$$

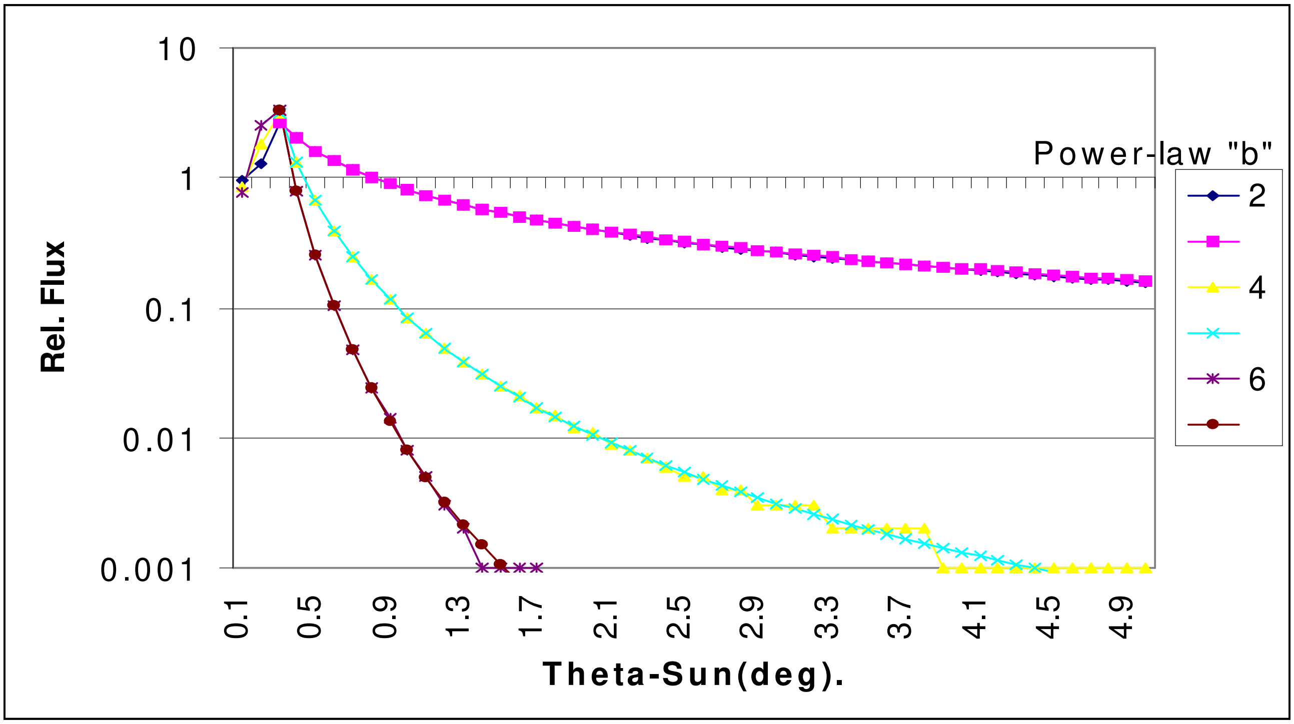
I was not able to analytically integrate eq. 2, so instead I have resorted to an analytic approximation to the integral, and a numerical integration.

One would expect the largest contribution to this integral to come from $l \cong L_*$ when r is the smallest. So, writing $l = L_*(1+\delta)$, $r^2 \cong 2L_*^2(1+\delta)(1-\cos\vartheta)$ which is a good approximation for $(L_*\delta)^2 << 2L_*^2(1+\delta)(1-\cos\vartheta)$. Using the approximation $(1-\cos\vartheta) \cong \vartheta^2/2$, which will always be a good approximation for the problem at hand, one can show that the limits on δ are between $\pm\vartheta$, which will be our limits of integration (note for $\vartheta<\vartheta_{sun}$, this is not a good approximation). The integral can then be performed with the result

$$\frac{dN\gamma}{d(\cos\vartheta)} = \frac{aAR_*^b}{L_*^{b-1}}\frac{1}{\vartheta^{b-1}} \qquad for\, b \neq 2$$

$$\frac{dN\gamma}{d(\cos\vartheta)} = \frac{aAR_*^b}{2L_*^{b-1}}\frac{1}{\vartheta^2}\ln\left[\frac{1+\vartheta}{1-\vartheta}\right] \approx \frac{aAR_*^b}{L_*^{b-1}\vartheta^{b-1}} \qquad for\, b = 2$$

The analytic calculation gives $dN_\gamma/d(\cos\vartheta) \propto 1/\vartheta^{(b-1)}$ for all values of b. This result can be checked with a precise numerical calculation, as shown in the following figure



It is fairly clear that, outside of the direction to the sun (≅0.25°), the numerical integration and the analytic approximation are in excellent agreement (you may not be able to see the differences of the two distributions that are overlaid on each other). The analytic result was normalized to the numerical result at the sun's position; the numerical results throughout are normalized to a flux of one unit at the center of the sun.

Since the analytic result is very accurate, I will use it to study the optimal bin size and signal efficiency. The optimal bin size is, as usual, that bin which optimizes the signal to sqrt(background), where as mentioned above I will be ignoring (for now) effects due to Milagro's angular resolution and the shape of the sun's shadow. (N.B. The shadowing due to the sun is an effect of at most $\sim(\vartheta_{sun}/\vartheta)^2$, where ϑ is the angular cut made around the sun's direction. It will decrease the background by at most this value because the heliomagnetic cutoff will lessen the size of the decrease, and the shift of the shadow's position w.r.t the sun's direction will also lessen the size of the decrease about the sun's position. We will find an optimal bin of $\sim 3\vartheta_{sun}$, which means this is ~10% reduction in the background level that must be accurately taken into account.) Letting the optimal be defined by the maximum of

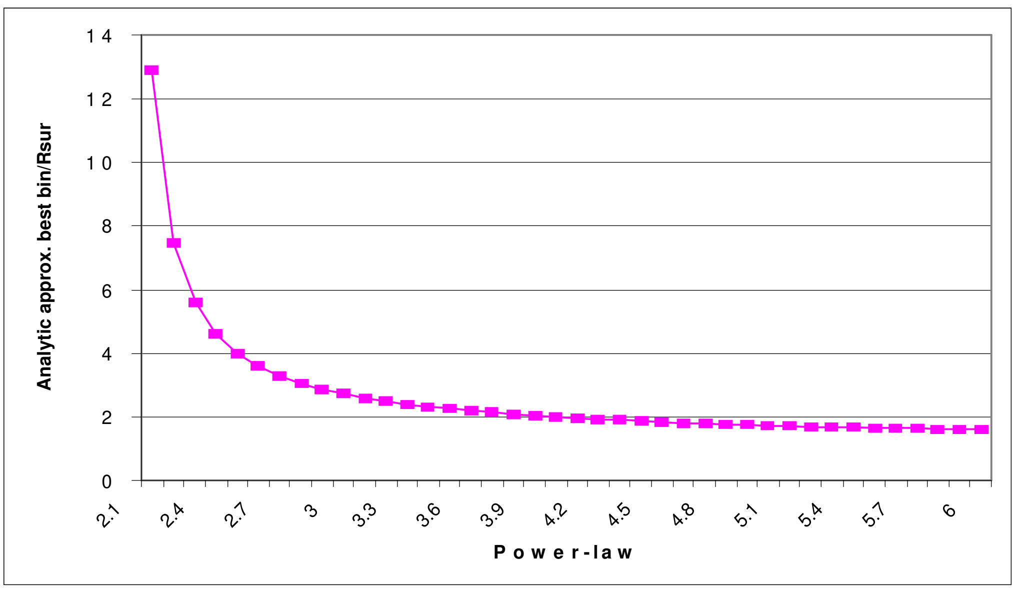
$$S = \frac{N(\vartheta)}{\sqrt{B(\vartheta)}} \propto \frac{\int_0^{\vartheta} \frac{\partial N_\gamma}{\partial \cos\vartheta} d\cos\vartheta}{\sqrt{1-\cos\vartheta}} \approx \frac{const * \left(\frac{1}{\vartheta_{sun}^{b-3}} - \frac{1}{\vartheta^{b-3}}\right)}{\left(\frac{\vartheta}{\sqrt{2}}\right)}$$

(for b ≠ 3). Solving for the maximum (minimum if b ≤ 2, which I will ignore from hereon), I find the optimal bin size to be

$$\vartheta_{opt} \cong \frac{\vartheta_{sun}}{(2-b)^{1/(3-b)}} \qquad b \neq 3$$

$$\vartheta_{opt} \cong e * \vartheta_{sun} \qquad b = 3$$

by solving $\partial N_\gamma/\partial\vartheta=0$. A graph of the optimal bin is shown here:



One fact may be very important to note here. That is that there may be a very large amount of the signal outside of the optimal bin. Typically we are concerned with the case where there is a moderate (~1/3) amount of signal just outside of the optimal bin. In this case, there may be much more. Using the analytic approximation, I find

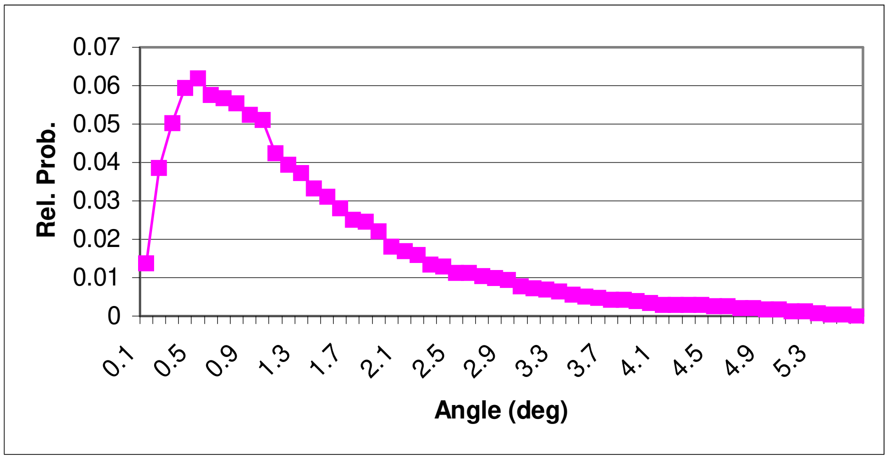
$$\frac{N(\vartheta_{opt})}{N(\infty)} \cong \frac{b-2}{b-1} \qquad b \neq 3$$

$$\frac{N(\vartheta_{opt})}{N(\infty)} \cong \frac{1}{\ln\left(\frac{"\infty"}{\vartheta_{sun}}\right)} \qquad b = 3$$

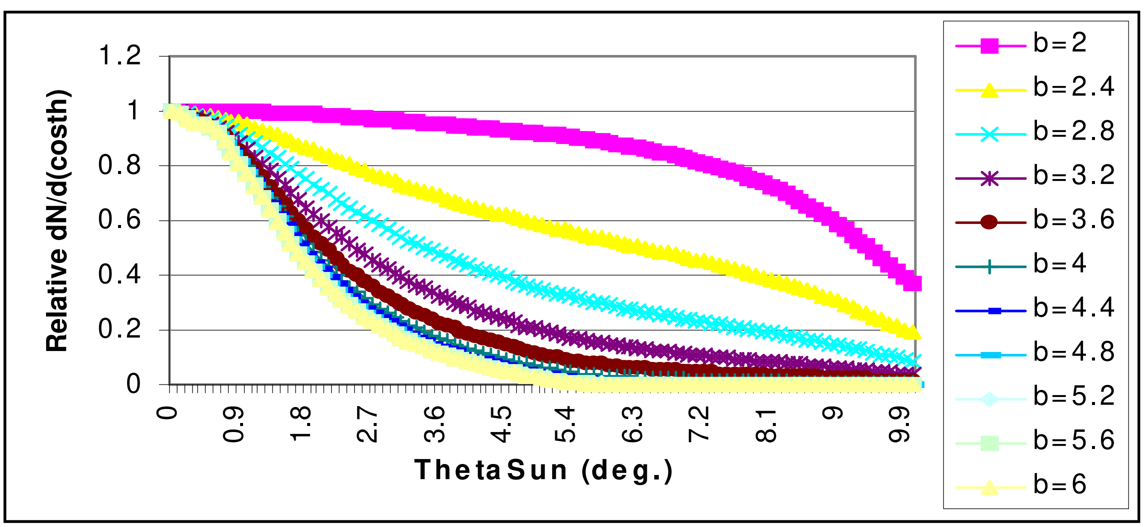
so that we may have ~50% or more of the signal extending well outside of the analysis region. This will make not only estimation of the true flux difficult, but perhaps more importantly the true background level may be very difficult to estimate because of the large extended signal "halo" of gamma-ray signal around the sun.

Angular Resolution Effects

To include the effects of finite angular resolution, I have resorted to a completely numerical calculation. As an example, I use the angular resolution appropriate for Milagrito, provided by J. McCullough, for N_{fit}>40 cut. The resolution function is shown here:



The long tail on the resolution function extends the size of the "halo" of the signal around the sun. Using this situation as an example, I show the expected distribution of signal around the Sun for various assumed power-law distributions in this figure



For the relatively flat power-laws $b \cong 2$, there is a great deal of signal for a large distance away from the Sun (note the cutoff at ~10 degrees is an artifact of how I cut the integration off). For $b>4$, this is not a large effect.

Summary

I have presented detailed analytic and numerical calculations of the shape of the gamma-ray signal around the Sun for a variety of models of the actual WIMP annihilation distribution. It is clear from these studies that the signal may be distributed over a large part of the sky. As pointed out in a previous memo, a sensitive search for a weak flux of gamma-rays is laden with potential systematic pitfalls in the background estimation method; the problem is compounded here by the extent of the signal region and the presence of other “signals” in the Solar neighborhood.