Deflection of Cosmic Rays in the Earth's Magnetic Field

Characterization of the Earth's Field

The earth's magnetic field can be approximated by a dipole with the north magnetic pole, which is a south pole, near the earth's North Pole. In actuality, the magnetic pole is located at approximately 78.6° N, 70° W. The field at the north magnetic pole (at sea level) is 0.572 Gauss. At the location of Milagro, 35° 52' 45" N, 106° 40' 37" W, the magnetic field is:

$$B_x = 0.229 \text{ Gauss (towards N)}$$
$$B_y = 0.044 \text{ Gauss (toward E)}$$
$$B_z = 0.464 \text{ Gauss (down).}$$

Note that the major component of the magnetic field is radial (down); the horizontal component of the field points somewhat east of north. It is convenient at this point to convert to a coordinate system where r is the (radial) direction towards the center of the earth, and λ is in the direction of the field component perpendicular to r.

In the dipole approximation, the field is given by:

$$B_r = -2\mu_o M \sin\theta_L \, [4\pi r^3]^{-1}$$
$$B_\lambda = -\mu_o M \cos\theta_L \, [4\pi r^3]^{-1},$$

where θ_L is the magnetic latitude (~43° for Milagro). Using the values of the magnetic field given above, and r_e = 6370 km, we find

$$\mu_o M = 1.1 \times 10^{12} \text{ Gauss - km}^3.$$

One might ask how good is the dipole approximation. In the following plot, I compare the dipole approximation with a detailed calculation of the field from the IGRF95 code at http://www.best.com/~williams/magvar.html. The approximation is actually not too bad! It is certainly good enough for a crude calculation of the magnetic deflection of cosmic rays.

Calculation of cosmic-ray deflection at zenith

In the approximation of small deflections, the change in direction of a charged particle of momentum p traversing a magnetic field is:

$$\Delta\theta = (Ze/p) \int B_\perp \, dL,$$

where Ze is the particle charge, $B_\perp$ is the component of the magnetic field perpendicular to the particle's momentum, and the integral is over the particle's trajectory. Putting in the correct units, we find

$$\Delta\theta = (0.03Z/p) \int B_\perp \, dL,$$

with B measured Gauss, L is measured in km, and p is measured in GeV/c.

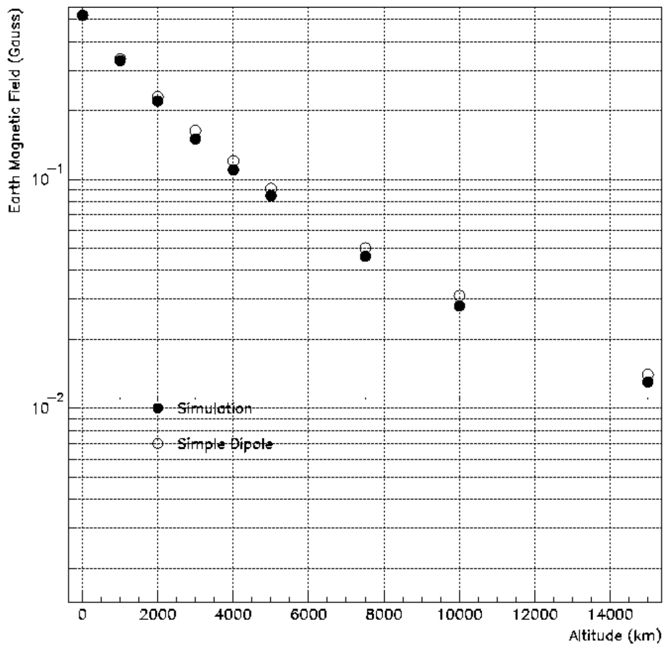

Figure 1. Comparison of the dipole approximation and a detailed model of the earth's magnetic field at the location of Milagro.

For vertically incident particles, $B_\perp = B_\lambda$, so we have:

$$\Delta\theta = (0.03Z/p)\int B_\lambda\, dL$$

$$= (0.03Z/p)\,(1.1 \times 10^{12}) \int \cos\theta_L\, [4\pi r^3]^{-1}\, dr$$

$$= (3.3 \times 10^{10})\,(Z/p)\,(0.73)\,[-8\pi r^2]^{-1}\Big|_r^\infty$$

$$= (9.6 \times 10^{8})\,(Z/p)\,[1/r^2]\,.$$

In the above equation, I assume that the particle momentum is always vertically downward and so it is valid only in the approximation that the magnetic deflection is small. Plugging in the earth's radius (6370 km), Z=1, and 1 TeV, I find:

$$\Delta\theta = 23.7 \text{ mr} = 1.4^\circ \text{ (towards the east).}$$

Note that if protons are bent towards the east, they appear to come from west of their actual origin when viewed from the earth, so the moon's shadow will be displaced to the west.

Cosmic-ray deflection from the direction of the moon

The above is correct for cosmic rays incident from the zenith. Of course, the moon never reaches the zenith here. One really should redo this calculation for actual incident directions pointing back to the moon. To do this, we first need to understand where the moon is in the sky as seen from Milagro. Andy ran a short program mapping the moon's apparent declination over time; the result is that the moon oscillates around the earth's equator with an amplitude of $\approx 19^{\circ}$ and a period of 28 days. Thus the moon's zenith angle at transit varies from $\sim 16^{\circ}$ to 56°, always towards the south. Considering the direction of the earth's field, it is apparent that this significantly reduces the magnetic deflection of cosmic rays. Particles incident from the south at a zenith angle of 26° (and the moon is at that location some of the time) enter along the magnetic field lines and suffer no magnetic deflection whatsoever! Below is a table of magnetic deflection of 1-TeV protons incident from the south (i. e. at transit) vs. zenith angle:

Declination	Zenith angle @ Transit	Deflection	
$+35.8^{\circ}$	0°	23.7 mr (E)	(moon doesn't get here)
$+20^{\circ}$	15.8°	10.7 mr (E)	
$+10^{\circ}$	25.8°	2 mr (E)	
0°	35.8°	-6.8 mr (W)	
-10°	45.8°	-14.6 mr (W)	
-20°	55.8°	-23.4 mr (W)	

On average, there is only a ~7 mr deflection towards the west (i. e. the shadow should appear to the east of the moon) for 1-TeV protons coming from the direction of the moon; this is <u>opposite</u> to the direction that a vertically incident proton is deflected (which is what we have assumed heretofore).

When the moon is to the east of transit, there is an additional deflection towards the south, while when the moon is west of transit, there is a deflection to the north. The size of this deflection is $\propto B_r \sin\theta_{EW}$, where θ_{EW} is the angle the incident cosmic ray direction makes with respect to due south. While this deflection can be substantial (certainly as large as several degrees for $\theta_{EW} \approx 30^{\circ}$), it averages out to zero over a day. We should improve the shape of the moon shadow by looking separately at events before and after transit.

Where does the deflection take place?

Finally, let's ask at what altitude does most of the bending take place. It is important to note that the dip angle of the geomagnetic field varies very little with altitude: 63.3° at sea level, 62.8^{o} at an altitude of 15,000 km. This means we can just look at the equation above without further complication. In doing this calculation, we must remember to cut off the integral at an altitude of ~20 km, where the first interaction takes place. The table below shows the bending between selected altitudes for a vertical, 1-TeV proton.

Starting Altitude	Ending Altitude	Bending Angle
1,000 km	20 km	6.6 mr
2,000 km	1,000 km	4.0 mr
3,000 km	2,000 km	2.8 mr
4,000 km	3,000 km	2.0 mr
5,000 km	4,000 km	1.5 mr
10,000 km	5,000 km	4.5 mr

Note that there is still 6 mr of bending above 10,000 km, 22% of the total.

Conclusions

Clearly, the apparent deflection of the moon's shadow in cosmic rays is complicated. The shadow is not simply deflected 24 mr to the west, as we had assumed. In fact, the E-W deflection depends on the declination of the moon, is quite a bit smaller than 24 mr, and changes sign within the moon's trajectory; it might prove to be difficult to observe the geomagnetic deflection of the shadow! Interestingly, there is a significant N-S deflection, that changes sign between data before transit and after transit of the moon. This effect cancels in the total data set, but it does lead to a smearing of the shadow. The data could be look at in a way to minimize this effect, or to maximize this effect; it might be the best place to look for the geomagnetic deflection.

The next step is to run a Monte Carlo tracking the moon's position to ascertain how best to look at the moon's shadow.