Cosmological Corrections and Calculating Intrinsic GRB Limits with Milagro

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ABSTRACT

Correcting limits for cosmological effects is non-trivial, and this memo details how limits should be calculated to incorporate cosmological effects.

1. What is Limited, and How to Calculate It

The first step in calculating a limit for a population of sources at moderate to high redshift (z), is to determine the observed redshift distribution of those sources. This incorporates three different factors: the comoving volume rate of the sources as a function of z (such as the star formation rate $\mathrm{M}_\odot/\mathrm{Mpc}^3/\mathrm{year}'$ where $'$ indicates comoving time), the comoving volume element as a function of z, and the observed time as a function of z. Integrating the source volume rate times $d$volume$/dz$ over dz and dt' yields the total number of sources one expects to observe. In the following paragraphs we will step through each of these terms independently.

The volume rate of sources is a predetermined function of z and depends on the model parameters of interest. For GRB limits we can use the popular assumption that GRBs follow the star formation rate (SFR) of the universe in comoving coordinates. The basic functional form is

$$\mathit{SFR} = C f(z), \tag{1}$$

where we have separated out a constant for later convenience. While the actual rate is currently a topic of hot debate, for this paper it is modeled as

$$C_{\mathit{SFR}} = \frac{1\ \mathrm{M}_\odot}{\mathrm{Mpc}^3\ \mathrm{year}'} \qquad f(z) = \begin{cases} 10^{1.45z-1.52} & 0 < z \leq 0.9 \\ 0.6095 & z > 0.9\ \ . \end{cases} \tag{2}$$

The total number of events one expects to see is then given by

$$\mathrm{Events} = \int_{z_1}^{z_2} \int_{t_1'(z)}^{t_2'(z)} \int_0^{\triangle\Omega} C f(z) \frac{dV(z)}{dz\, d\Omega}\, d\Omega\, dt'\, dz. \tag{3}$$

Of particular note in the previous equation is that the time integral is in comoving years, not observer years. This can be simplified using the relation $t = (1+z)t'$, and letting $\triangle t$ be the length of observation at the earth. We then obtain

$$\begin{aligned} \text{Events} &= \int_{z_1}^{z_2} \int_0^{\triangle t/(1+z)} \int_0^{\triangle \Omega} C f(z) \frac{dV(z)}{dz\, d\Omega}\, d\Omega\, dt'\, dz & (4) \\ &= C \triangle t \triangle \Omega \int_{z_1}^{z_2} \frac{f(z)}{(1+z)} \frac{dV(z)}{dz\, d\Omega}\, dz. & (5) \end{aligned}$$

Equation 5 is the key integral which needs to be solved for a number of different parameters to determine our upper limits. The excellent paper by David W. Hogg astro-ph/9905116 discusses cosmological corrections and gives relationships which can be used for $dV(z)/dz\, d\Omega$. The integral can then be evaluated numerically.

Since we expect GRBs to follow the star formation rate (for these limits), we can use $f(z)$ from Equation 2 but a different constant C to determine the expected number of events. The Monte Carlo simulation which generates artificial GRBs for the analysis distributes the events over $0 < z \leq 1.28$ using

$$\frac{f(z)}{(1+z)} \frac{dV(z)}{dz\, d\Omega}\, dz, \tag{6}$$

the argument to the integral in Equation 5. Since the Monte Carlo uses the expected z distribution of events, the probability of Milagro identifying a VHE event which follows the SFR within the z range of interest is given by simply dividing the number of GRBs detected by the search routine by the number of artificial GRBs generated. Alternatively, the total probability of detection can be determined by weighting the probabilities listed in Table 1 of the GRB limit paper by the percentage of the overall rate (from Equation 5) which is expected in the given z bin, and summing to obtain the total detection probability.

The total detection probability can be used to place a direct constraint on the constant C. The expected number of detections D is equal to the expected number of events times the detection probability P,

$$D = P \times Events. \tag{7}$$

For no observations a 90% confidence limit requires that the expected number of detections must be ≤ 2.3. Substituting and simplifying gives

$$C = \frac{2.3\ P^{-1}}{\triangle t \triangle \Omega \int_{z_1}^{z_2} \frac{f(z)}{(1+z)} \frac{dV(z)}{dz\, d\Omega}\, dz}, \tag{8}$$

where $\triangle t$ is the length of observation and $\triangle \Omega$ is the field of view of the detector.[1] This gives the value C which is the actual limit that we can place with Milagro observations.

[1]Equation 8 is strictly only correct if $z_1 < z < z_2$ includes the full range of z over which events can be

2. Presenting Limits

Unfortunately, C is a somewhat awkward and anti-intuitive value, and people get upset if this is quoted (as evidenced by the collaboration outrage when I used this value at the New York collaboration meeting). So we need to translate the value C into some other limit which is more palatable.

The easiest number to use is the fraction of the star formation rate C/C_{SFR}. This value accurately conveys what limits we can actually place, and is probably the most useful value for a theorist. Experimentalists, however, tend to feel that this is too model dependent and unphysical.

The suggested alternative which has been used in several versions of the paper is to convert this number to Events/Gpc^3/year (where year is in the observer frame). This is done by plugging the new value of C into Equation 5 to obtain the expected number of events in one observer year in 4π sr over some nominal z range, and dividing by the total volume for that z. The problem with this value is that it depends strongly on the z range chosen since the event rate is not constant. While a more "physical" measurement, it also hides some of the model dependence which is inherently part of our upper limit.

I do not know what the best presentation method is. The limit is determined by following the calculation in part one, and the collaboration should discuss how this limit can be best presented.

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detected, and the range of z used to calculate P is also $z_1 < z < z_2$.