Milagro Memo

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Searching for Delayed Emission from a GRB: Search Strategies and Trials Factors

Abstract: There are many possible strategies that may be used to search for delayed (or prior) emission correlated with a detected burst. In general there are two parameters used in a search: the duration of the emission and the time distribution of the emission. The possible strategies differ in the choice of timescales to be searched for emission, the choices of burst duration to search, and finally the ordering of the search. If a particular model is being tested one may order the search to have maximum sensitivity for the most likely choice of model parameters. In the absence of a model, the optimal ordering of the search is ambiguous and depends on the bias of the observer. In the worst-case (typical) scenario one simply examines a large set of time series with various bin widths. If one finds something that appears significant one is left with the unenviable task of a posteriori assigning a probability to the observed feature. Here I will investigate four possible search strategies and apply them to the case of the observed features in the light-curve from GRB 970417a.

Search Strategies: In the discussion below I will ignore the complications associated with over sampling – both in burst duration and burst start time. Obviously a real search will over sample in both parameters. This is straightforward and has been discussed in detail elsewhere.

In devising a search strategy one of the questions is "Over what period of time should we search for emission?". One can either, define a search window and look for emission over various durations within this window or one can start at the original burst time and search until one finds a "significant" feature. The first two strategies outlined will utilize the "search until" strategy; the last two will utilize the predefined window strategy. For simplicity the search until strategy will be terminated at the end of the same predefined window used in the other strategy.

The Data: The data consists of a time series of events obtained around some time T_0 that have been binned into a set of histograms (each histogram has α bins, where α may differ from histogram to histogram). In addition the search window may also differ from histogram to histogram. Thus, when searching for short duration emission one may want to only search for a short period of time around T_0 and when looking for longer duration emission search over a larger time window. Keeping the number of bins constant while changing the bin width would yield uniform sensitivity to any burst duration, but may unduly limit the search window for short duration bursts. The histograms are numbered as follows:

$H_1 = \{b_1^{-N_1/2}, b_1^{-N_1/2+1}, \ldots, b_1^0, b_1^1, \ldots, b_1^{N_1/2}\}$
$H_2 = \{b_2^{-N_2/2}, b_2^{-N_2/2+1}, \ldots, b_2^0, b_2^1, \ldots, b_2^{N_2/2}\}$
.
.
.
$H_z = \{b_z^{-N_z/2}, b_z^{-N_z/2+1}, \ldots, b_z^0, b_z^1, \ldots, b_z^{N_z/2}\}$

The ordering of the histograms is such that H_1 has the smallest bin width and H_z has the largest bin width. The different search strategies can now be viewed as different orderings of the above set of b_α^j's. We are free to search them in any order we please. What one observer may consider the "natural" search order is simply a bias. The idea is similar to the playing of blackjack, when you believe the odds are in your favor you increase the size of your bet. Here, you order the bins in a manner consistent with your estimate of the likelihood of observing a signal. This bias may be based on a model but it must be believable. For example if we observe a feature in a random bin, we can concoct a scheme where this is the first bin searched and claim to have observed an improbable result. If we are attempting to establish an a posteriori significance to a feature we must be careful to avoid this trap.

In the examples below I assume that the time window searched is the same for all of the histograms, so that the number of bins in each histogram is inversely proportional to the bin width in the histogram.

Strategy 1: This is a "search until success" strategy. Here the plan is to burn our first trials on the largest bin widths. The rationale being that if we observe a marginal signal in one of the big bins it will not be significant if we have already performed the searches of the small bins. This strategy maybe used if we believe we have no knowledge of when a burst will occur or how long a burst last. The order of the search is:
$H_z : \{b_z^1, b_z^{-1}, b_z^2, b_z^{-2}, \ldots, b_z^{N_z/2}, b_z^{-N_z/2}\}$
$H_{z-1} : \{b_{z-1}^1, b_{z-1}^{-1}, b_{z-1}^2, b_{z-1}^{-2}, \ldots, b_{z-1}^{N_{z-1}/2}, b_{z-1}^{-N_{z-1}/2}\}$
.
.
.
$H_1 : \{b_1^1, b_1^{-1}, b_1^2, b_1^{-2}, \ldots, b_1^{N_1/2}, b_1^{-N_1/2}\}$

When I observe a bin whose probability is less than a predefined post-trial threshold, P, I terminate the search.

Strategy 2: This is also a "search until success" strategy. Here the plan is to burn our first trials on the bins closest in time to the original burst. Here we believe we know nothing about the expected duration of a burst but expect such a burst to be "near" the original burst. The order of the search is:
$\{b_1^1, b_1^{-1}, b_2^1, b_2^{-1}, \ldots, b_{z-1}^1, b_{z-1}^{-1}, b_z^1, b_z^{-1}\}$
$\{b_1^2, b_1^{-2}, b_2^2, b_2^{-2}, \ldots, b_{z-1}^2, b_{z-1}^{-2}, b_z^2, b_z^{-2}\}$
.
.
.
$\{b_1^{N/2}, b_1^{-N/2}, b_2^{N/2}, b_2^{-N/2}, \ldots, b_{z-1}^{N/2}, b_{z-1}^{-N/2}, b_z^{N/2}, b_z^{-N/2}\}$

Again we terminate the search if a predefine threshold has been observed. Note that we may have run out of bins to examine for the histograms with the larger bin widths.

This strategy is my personal favorite as it allows me to simultaneously search many burst duration time-scales and accumulates trials in each time-scale in a "natural" manner. The shorter duration the burst the closer it should be to the original burst. Given the statistics it makes little sense to search for 1 second bursts associated with the original burst days or weeks after the original burst, but it may well make sense to search for day-long bursts weeks after the original burst.

Strategy 3: Here we look at everything and determine the post-trials probability of the most significant feature observed. However, we distinguish between the different durations (bin widths). This allows us to search for longer burst durations without burning all of our trials on the short duration bursts. We have no particular search order here. We simply examine all of the histograms and select the most significant (post-trial) feature b_α^j. If all of the bins were independent the final probability would be: $zP(b_\alpha^j)N_\alpha$. Where z is the number of independent histograms and N_α is the number of independent bins in the selected histogram.

Strategy 4: This is the insensitive approach. Everything is thrown together and all bins are treated equally. We select the most significant (pretrial/post-trial) feature b_α^j. The final probability is then $P(b_a{}^j)\sum_{i=1}^{z} N_i$.

An Example: I will attempt to compute the post-trials probability of the after bursts from GRB 970417a using each of the above strategies. In Figure 1 I show the light curves from my memo which first discussed these bursts http://scipp.ucsc.edu/milagro/memos/sinnis051399/index.html. Note that the bin size and position are different from that reported by Julie and Andy, so the pretrial probability of the 1st after burst is $2x10^{-5}$ (nObs = 17, nExp = 5.028). We immediately encounter a problem when trying to establish a final probability for the first two search strategies. In both of these strategies we must predefine a threshold after which we terminate the search. If we set that threshold a posteriori to be equal to the probability of the most significant feature in the histograms then we have "cheated" and underestimated our trials factor. Nevertheless I will assume that we predefined a threshold of 2% (post trials). We also have a choice of searching only for post-burst emission or searching for either post or pre-burst emission. Here I will assume that we are searching for both pre and post-burst emission.

The simulation: Each "experiment" consists a time series of Poisson data generated at a rate of 0.675 Hz (the background rate in the angular bin of interest, see the GRB paper for details). I then binned the data into 5 histograms with bin widths of 1, 2, 4, 8, and 16 seconds (as I had original done). The histograms were searched for significant features. The post-trials probability is then just the number of experiments in which the observed probabilities were surpassed divided by the total number of experiments run.

Results: For strategies 1 and 2 I define my threshold to be the pre-trial probability times the number of trials taken to that point (for each of the two bursts). Thus, for each burst and strategy the threshold is different. Each is tuned to give the minimum possible post-trials probability. This is not meant to represent the "true" probability of each burst, but to give a range of possible probabilities. I ran 100,000 experiments and counted the number of experiments in which this threshold was exceeded. In Table 1 I give the final probability resulting from each of the search strategies.

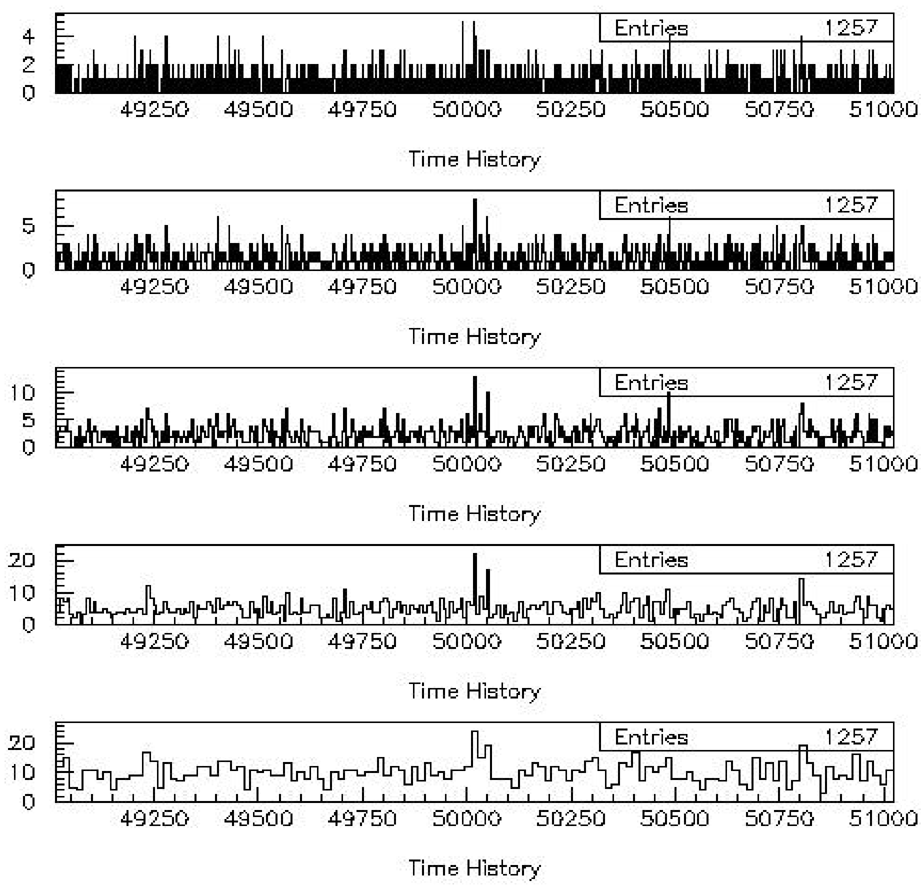

Figure 1: Light curve of GRB 970417a bin widths are (top to bottom) 1, 2, 4, 8, and 16 seconds.

Strategy/Burst	Probability (1)	Probability (2)
1/ Burst 1	1.2 %	25 %
1/ Burst 2	65 %	51 %
2/ Burst 1	0.16 %	2.8 %
2/ Burst 2	55 %	43 %
3/ Burst 1	1.1 %	33 %
3/ Burst 2	23 %	23 %
4/ Burst 1	2.0 %	34 %
4/ Burst 2	26 %	26 %

Table 1: Final probabilities for the 4 search strategies. Probability 1 assumes a pretrial probability of 2×10^{-5} for the first burst and 7×10^{-4} for the second burst (as originally observed). Probability 2 assumes a pre-trial probability of 5×10^{-4} for both bursts (as given in the current draft of the paper).

The first after-burst presented in the paper is marginally significant under one of the strategies and the second after-burst reported is significant under any of the strategies investigated. Of course there are strategies, which I consider "unnatural", under which the second burst would have the same significance as the first burst. The burst originally reported in my memo is marginally significant under all of the strategies investigated here, but the emphasis is on the word marginal. Perhaps a more complete analysis with over-sampling in space, time, and bin width will turn up a significant feature in our data. At this point however, it appears that we have little evidence with which to claim post burst emission.

Conclusions:

I have explored several possible search strategies and investigated the effect of strategy on probability for the after bursts observed from GRB 970417a. The choice of a search strategy has a strong effect on the post-trial probability. Clearly this can lead to a psychological trap when choosing a strategy after a feature has been observed in the data set. There are two ways out of this dilemma, the method which gives one maximum sensitivity to a burst that behaves as one's bias thinks it should, is to define the search criteria before looking at the data. However, this has the down side that it is almost impossible to contain our natural curiosity, we as a collaboration would have to agree on the strategy, and one must convince the community that the search strategy really was defined a priori. The second method is to require such a large signal that it would be significant no matter what search method was employed. Not only is this method the easiest to enforce, it has the added advantage of believability, especially necessary when one is claiming to have discovered a new phenomenon.