Milagrito's Point Spread Function and Mrk 501

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Abstract. This memo describes a re-analysis of the Mrk 501 data from Milagrito using a binned maximum likelihood method [1] without cuts on the number of tubes in the angle fit. The point spread function is assumed to be a superposition of two Gaussian distributions. The analysis is performed both for a fixed source position *and* with the source position as a free parameter. The significance at the source position is $3.7\,\sigma$ and thus comparable to the standard analysis, and no indication of a pointing error is found.

1. Introduction

In the standard binned analysis of Mrk 501, a significance $> 3\,\sigma$ is only achieved after removing all events with $n_{fit} < 40$, where n_{fit} is the number of tubes remaining in the standard angle fit (version 23). This rather strong cut (close to 50% of the events are lost) is necessary if we choose an analysis with a fixed source bin size that treats all events equally. However, Milagrito's – and subsequently Milagro's – point spread function (PSF) varies heavily from event to event, and a more appropriate point source analysis would treat every event according to its quality. The analysis presented here follows the binned EGRET point source analysis [1] using a maximum likelihood ratio test. This analysis is "binned" in the sense that the events are not analyzed one by one, but as a sky map with (sufficiently small) bins in right ascension and declination in a (sufficiently large) region of the sky around the potential source. To take care of the dependence of the PSF on parameters like n_{fit}, the data is also binned in different categories with different PSFs.

2. The Milagrito Point Spread Function

The PSF of Milagrito has a sharper core and wider tails than a Gaussian distribution and varies significantly as a function of the event size n_{fit}. The PSF can in principle be derived from simulated showers or from the data by using $\Delta_{EO}/2$, where Δ_{EO} is the space angle difference between the shower direction as seen by two independent, interleaved parts of the detector. Δ_{EO} is not sensitive to certain systematic errors, thus a direct approach using the space angle Δ_θ between the true and the fitted shower direction in the simulation is preferable (if one neglects uncertainties in the simulation for now). However, the goal of this analysis is to take into account the rapid change of the PSF as a function of the event size, which requires splitting the data into a sufficiently large number of categories, and the PSF has to be determined for each category. The total number of simulated showers available is not large enough for

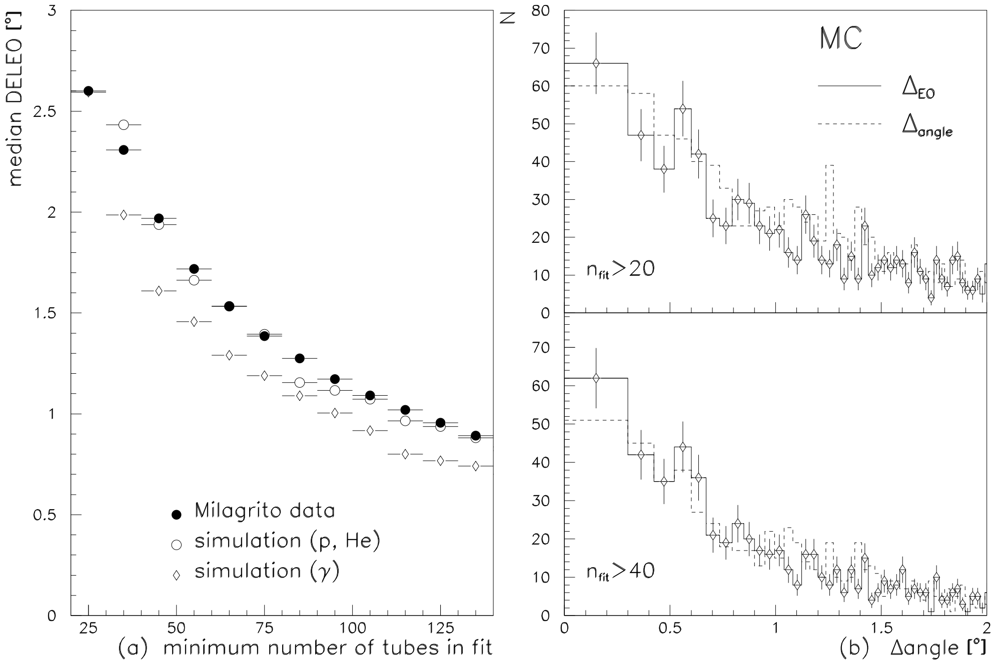

Figure 1. (a) Δ_{EO} as a function of the number of tubes remaining in the shower plane fit for simulation and data. (b) Distribution for $\Delta_{EO}/2$ and Δ_θ for two different cuts in n_{fit}.

this, so consequently, Milagrito *data* is used. In order to justify this approach, some checks are required: we have to (1) compare Δ_{EO} as a function of the n_{fit} for data and simulation and (2) to compare $\Delta_{EO}/2$ and Δ_θ for the simulation.

Figure 1 (a) shows Δ_{EO} for Milagrito data and for MC γ- and cosmic ray showers and indicates that there is no major discrepancy between data and simulation. The difference between γ-showers and hadron showers in the simulation can be explained by the different core distribution of these shower types: there is no difference evident if only showers with core on (off) the pond are compared. As Milagrito has limited capability of locating the core, splitting the data sample into on and off pond events is not possible and the PSF derived from data will overestimate the width of the true γ-shower PSF by about $10\% - 20\%$. In contrast to this systematic error that leads to an overestimation of the width of the PSF, the use of $\Delta_{EO}/2$ instead of Δ_θ may result in an underestimation of the width:

Figure 1 (b) shows the distribution for Δ_{EO} and Δ_θ for two different cuts in n_{fit}. Although the difference between the distributions is (unfortunately) well within the statistical error, the Δ_{EO} distribution tends to be slightly sharper. Especially for events with small n_{fit} and events with large n_{fit}, we expect these distributions to deviate considerably.

Rather then correcting for the known shortcomings of this approach, we use the PSF as estimated from the data and check how crucially the performance of this

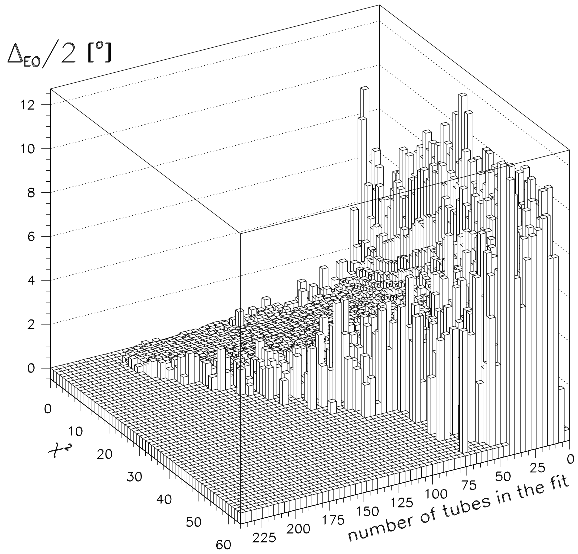

Figure 2. DELEO/2. as a function of the number of tubes remaining in the shower plane fit and the χ^2 of the angle fit.

analysis method on Mrk 501 data depends on the actual choice of the width of the Gaussian distributions.

For a fixed n_{fit}, the Δ_{EO} distribution also shows a dependence on the χ^2 of the angle fit (see Figure 2). Following David Williams, the analysis categorizes the data according to n_{fit} and χ^2 in 17 bins † with decreasing median of $\Delta_{EO}/2.355$. The upper values for the bins are 0.35°, 0.45°, 0.55°, 0.65°, 0.75°, 0.85°, 0.95°, 1.1°, 1.3°, 1.5°, 1.7°, 1.9°, 2.1°, 2.3°, 2.5°, 2.9°, ∞. The PSFs for each of these categories are approximated by fitting the $\Delta_{EO}/2$ distribution to

$$PSF_\theta = \sum_{i=1}^{2} A_i e^{-B_i\theta^2}, \tag{1}$$

with

$$B_i = \frac{1}{2\theta_i^2}\ , \qquad \theta_1 < \theta_2\ . \tag{2}$$

This choice takes sufficient care of the narrow central portion and the tail of the distribution (EGRET uses a sum of 4 Gaussian distributions [2]). The resulting values for θ_1 and θ_2 together with the result of the fit for two typical categories are shown in Figure 3. For the last three categories, a fit to a *single* Gaussian distribution gives reasonably good results, so only θ_1 appears in the plot. As θ_1 and θ_2 increase, the relative contribution of the Gaussian distribution with the largest width, θ_2, becomes more dominant as more events are in the tail of the distribution; the ratio $A_2/(A_1+A_2)$ increases from 0.1 (category 1) to about 0.5 (category 14).

† The first 3 ("best") bins of the original classification are combined in the following analysis; the value of θ_1 thus does not drop below 0.3°, as was the case in the Gaussian weighting analysis.

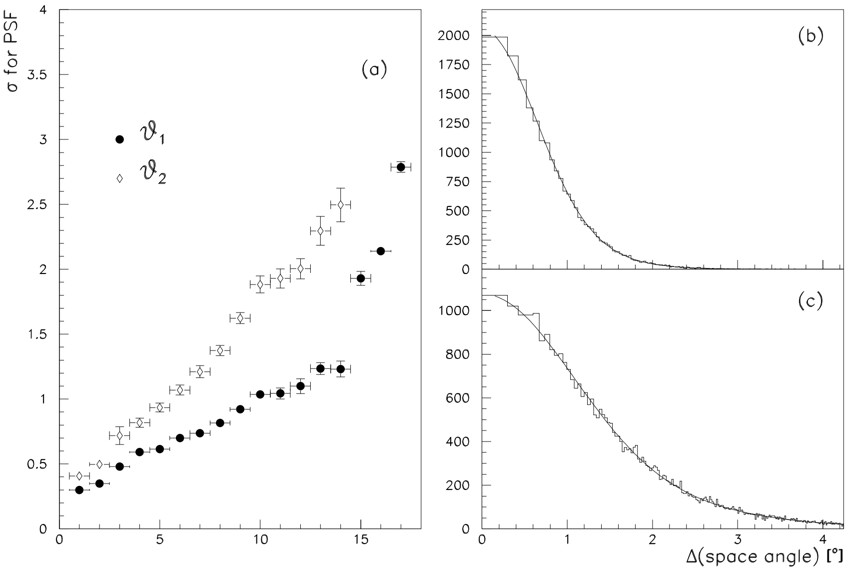

Figure 3. (a) PSF parameters θ_1 and θ_2 for the 17 categories. (c) and (d) show fits of PSF_θ to typical categories.

3. Likelihood Analysis

For the Milagrito analysis, a $20^\circ \times 20^\circ$ region of the sky with the Mrk 501 position in the center is used as the region of interest. This region is binned in 201×201 bins with a bin size of $0.1^\circ \times 0.1^\circ$. The likelihood is the product of the probability for each pixel,

$$L = \prod_{ij} p_{ij} \ , \tag{3}$$

where

$$p_{ij} = \frac{\theta_{ij}^{n_{ij}} \ e^{-\theta_{ij}}}{n_{ij}!} \tag{4}$$

is the Poisson probability of observing n_{ij} counts in pixel ij when the predicted number of counts is θ_{ij}. The logarithm of the likelihood is

$$\mathrm{lnL} = \sum_{\mathrm{ij}} \mathrm{n_{ij}} \ln(\theta_{\mathrm{ij}}) - \sum_{\mathrm{ij}} \theta_{\mathrm{ij}}, \tag{5}$$

where the model independent term $\sum \ln(\mathrm{n_{ij}}!)$ is neglected.

The model prediction for pixel ij is

$$\theta_{ij} = b_{ij} + N_s PSF(\alpha, \delta, n_{fit}, \chi^2, i, j) \tag{6}$$

where b_{ij} is the expected background for pixel ij, N_s is the number of counts from the source located at right ascension α and declination δ, and $PSF(\alpha, \delta, n_{fit}, \chi^2, i, j)$

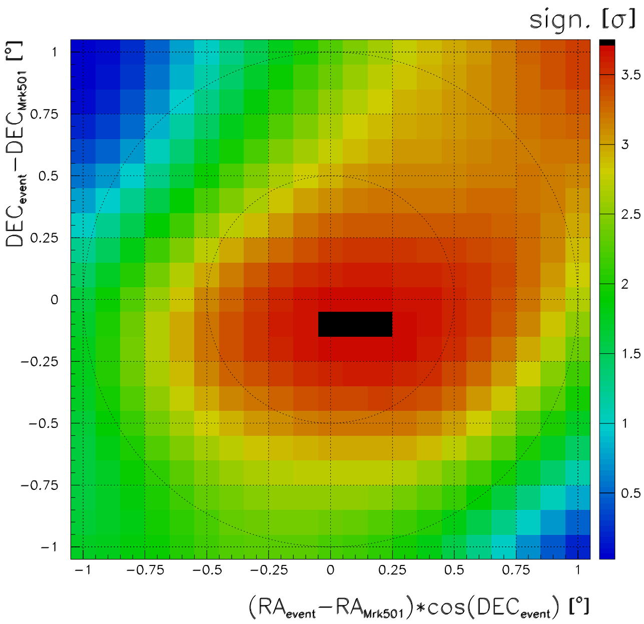

Figure 4. The significance for an event excess as a function of right ascension (RA) and declination (DEC) in a $2^o \times 2^o$ region with the Mrk 501 position in the center.

is the fraction of the PSF located at (α, δ) that is in pixel ij (Equation 6 of course assumes that there is no other source in the same search region, unfortunately a safe assumption for Milagrito). N_s and, in order to check for systematic pointing errors, also the source position (α, δ) are subject to parameter estimation by a likelihood ratio test; "there is no source at (α, δ)", *i.e.* $N_s = 0$ is the null hypothesis. The logarithm of the likelihood ratio is $\ln L_0 - \ln L$, and $-2(\ln L_0 - \ln L)$ is expected to follow a χ^2-distribution with one degree of freedom in the null hypothesis. As the number of counts is rather large for point source searches in the entire Milagrito data set, the expected deviation from a χ^2-distribution should be negligible - an assumption that is currently under investigation.

The significance of the observation is $S = \sqrt{-2(\ln L_0 - \ln L)}$.

4. Analysis of Mrk 501 data

The binned likelihood method was used to analyze the data set taken on Mrk 501 between February 1997 and April 1998. No cut on n_{fit} was applied. For a fixed source position with $\alpha = 253.468^o$ and $\delta = 39.760^o$, the analysis gives a maximum likelihood for $N_s = 7622$, corresponding to a significance of $3.7\,\sigma$, which is about the same significance as for the standard analysis with a cut $n_{fit} > 40$ and 3624 excess events. While we did not "win" in terms of significance, the likelihood analysis with different PSF categories makes a cut on n_{fit} unnecessary and does not require the search for an optimal source bin size.

In order to check whether there is evidence for a systematic pointing error, N_s *and* the source position (α, δ) are treated as free parameters. The maximum significance

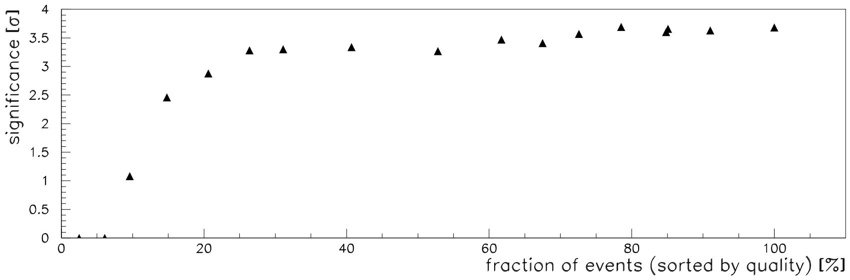

Figure 5. The significance for an event excess as a function of the fraction of events – starting with the "best" events – included in the analysis.

is found about $0.25°$ away from the the true source position and thus well within Milagrito's "error box". Figure 4 shows the significance as a function of right ascension and declination in a $2° \times 2°$ region with the Mrk 501 position in the center.

It is noteworthy that an analysis using a properly normalized step function

$$PSF_\theta = \begin{cases} \text{const.} & \text{if } \theta < 1.0° \\ 0. & \text{otherwise} \end{cases} \tag{7}$$

which approximately simulates a binned analysis with a $1.0°$ bin size roughly reproduces the $\sim 2\,\sigma$ excess of the standard analysis with no cut on n_{fit}.

The excess significance turns out to be rather stable: even if we assume that the width of the Gaussian distributions has not been estimated correctly by using Δ_{EO}, the significance of the excess does not change significantly when θ_1 and θ_2 are changed within reasonable limits. As expected, a correct estimation of the PSF for the "best" events is much more crucial than for the categories with worse resolution, where a change in θ_i almost does not affect the final result at all. Unfortunately, the "best" categories are probably also likely candidates for an underestimated width of the Gaussian distributions. However, even assuming that the PSF is more than 30% wider than estimated from Δ_{EO} does not lead to a change in significance of more than 10% - in fact, the excess significance slightly increases with a broader PSF for the "best" categories.

Figure 5 shows the significance as a function of the fraction of the events included in the analysis, starting with the "best" category only and subsequently adding categories. The best 30% of the showers give a total significance of about $3.3\,\sigma$, which is almost as good as the final result. The "worst" 20% of the data do not add to the significance any more – in this type of analysis, they also do not decrease the significance.

References

[1] J.R. Mattox et al., *Astrophys. J.* **461**, 396 (1996).
[2] D.J. Thompson et al., *Astrophys. J. Suppl.* **86**, 629 (1993).