The Big Idea

Acceleration is caused by force. All forces come in pairs because they arise in the interaction of two objects — you can't hit without being hit back! The more force applied, the greater the acceleration that is produced. Objects with high masses are difficult to accelerate without a large force. In the absence of applied forces, objects simply keep moving at whatever speed they are already going. In formal language[1]:

Newton's 1st Law: *Every body continues in its state of rest, or of uniform motion in a right* (straight) *line, unless it is compelled to change that state by forces impressed upon it.*

Newton's 2nd Law: *The change of motion is proportional to the motive force impressed; and is made in the direction of the right* (straight) *line in which that force is impressed.*

Newton's 3rd Law: *To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.*

Key Concepts

- An object will not change its state of motion (i.e., accelerate) unless an unbalanced force acts on it. Equal and oppositely directed forces do not produce acceleration.
- If no unbalanced force acts on an object the object remains at constant velocity or at rest.
- The force of gravity is called weight and equals m**g**, where **g** is the acceleration due to gravity of the planet (**g** = 9.8 m/s^2, downward, on Earth).
- Your mass does not change when you move to other planets, because mass is a measure of how much *matter* your body contains, and not how much gravitational force you feel.
- To calculate the net force on an object, you need to calculate all the individual forces acting on the object and then add them as vectors. This requires some mathematical skill.
- Newton's 3rd Law states for every force there is an equal but opposite reaction force. To distinguish a third law pair from merely oppositely directed pairs is difficult but very important. Third law pairs must obey three rules: they must be of the same type of force,

[1] Principia in modern English, Isaac Newton, University of California Press, 1934

they are exerted on two different objects and they are equal in magnitude and oppositely directed. *Example:* A block sits on a table. The Earth's gravity on the block and the force of the table on the block are equal and opposite. **But these are not third law pairs,** because they are both on the same object and the forces are of different types. The proper third law pairs are: (1) earth's gravity on block/block's gravity on earth and (2) table pushes on block/ block pushes on table.

- If you're asked to evaluate a vector, you may state the x and y components of the vector, or a magnitude and an angle with respect to the horizontal.

Key Equations

- $\mathbf{a} = \mathbf{F}_{net} / m$; the acceleration produced depends on the net force on an object and its mass.
- $\mathbf{F}_{net} = \sum \mathbf{F}_{\text{individual forces}} = m\mathbf{a}$; the net force is the vector sum of all the forces acting on the object.
- $F_{net,x} = \sum F_{\text{x-direction forces}} = ma_x$; the net force in the x-direction is the sum of all the forces acting on the object in the x-direction.
- $F_{net,y} = \sum F_{\text{y-direction forces}} = ma_y$; as above, but in the y-direction.
- $\mathbf{F}_g = m\mathbf{g}$; the force of gravity acting on an object, often simply called the "weight" of the object. On Earth, $\mathbf{g} = 9.8$ m/s^2 in the downward direction.
- $\mathbf{F}_N$; the normal force is a contact force that acts in a perpendicular direction to a surface.[2]
- $\mathbf{F}_{sp} = -k(\Delta\mathbf{x})$; the spring force is the force a coiled spring exerts when it is either compressed or expanded by a displacement $\Delta\mathbf{x}$ from its resting position.[2] The spring constant k depends on the strength of the spring, and has units of N/m.
- $\mathbf{F}_T$; the force of tension is a force that acts in strings, wires, ropes, and other non-stretchable lines of material.[2]
- $f_s \leq \mu_s F_N$; the force of static friction acts between two surfaces that are in contact but not in motion with respect to each other. This force prevents objects from sliding. It always opposes potential motion, and it rises in magnitude to a maximum value given by this formula.[2]
- $f_k = \mu_k F_N$; the force of kinetic friction acts between two surfaces that are in contact and in motion with respect to each other. This force slows sliding objects. It always opposes motion.[2]

[2] Ultimately, many of these "contact" forces are due to attractive and repulsive electromagnetic forces between atoms in materials.

Problem Solving for Newton's Laws, Step-By-Step

1. *Figure out which object is "of interest."*

a. If you're looking for the motion of a rolling cart, the cart is the object of interest.
b. If the object of interest is not moving, that's OK, don't panic yet.
c. Draw a sketch! This may help you sort out which object is which in your problem.

2. *Identify all the forces acting on the object and draw them on object.(This is a free-body diagram --FBD)*

a. If the object has mass and is near the Earth, the easiest (and therefore first) force to write down is the force of gravity, pointing downward, with value *mg*.
b. If the object is in contact with a flat surface, it means there is a normal force acting on the object. This normal force points *away* from and is perpendicular to the surface.
c. There may be more than one normal force acting on an object. For instance, if you have a bologna sandwich, remember that the slice of bologna feels normal forces from both the slices of bread!
d. If a rope, wire, or cord is pulling on the object in question, you've found yourself a tension force. The direction of this force is in the same direction that the rope is pulling.
e. Don't worry about any forces acting on other objects. For instance, if you have a bologna sandwich as your object of interest, and you're thinking about the forces acting on the slice of bologna, don't worry about the force of gravity acting on either piece of bread.
f. Remember that Newton's 3rd Law, calling for "equal and opposite forces," does not apply to a single object. None of your forces should be "equal and opposite" on the same object in the sense of Newton's 3rd Law. Third law pairs act on two different objects.
g. Recall that scales (like a bathroom scale you weigh yourself on) read out the normal force acting on you, not your weight. If you are at rest on the scale, the normal force equals your weight. If you are accelerating up or down, the normal force had better be higher or lower than your weight, or you won't have an unbalanced force to accelerate you.
h. Never include "$m\mathbf{a}$" as a force acting on an object. "$m\mathbf{a}$" is the *result* for which the net force $\mathbf{F}_{net}$ is the *cause*.

3. *Identify which forces are in the* x*-direction, which are in the* y*-direction, and which are at an angle.*

a. If a force is upward, make it in the *y*-direction and give it a positive sign. If it is downward, make it in the *y*-direction and give it a negative sign.
b. Same thing applies for right vs. left in the *x*-direction. Make rightward forces positive.
c. If forces are at an angle, draw them at an angle. A great example is that when a dog on a leash runs ahead, pulling you along, it's pulling both forward *and* down on your hand.
d. Draw the free body diagram (FBD).
e. Remember that the FBD is supposed to be helping you with your problem. For instance, if you forget a force, it'll be really obvious on your FBD.

4. *Break the forces that are at angles into their* x *and* y *components.*

a. Use right triangle trigonometry
b. Remember that these components aren't *new* forces, but are just what makes up the forces you've already identified.

c. Consider making a second FBD to do this component work, so that your first FBD doesn't get too messy.

5. *Add up all the* x-*forces and* x-*components.*

a. Remember that all the rightward forces add with a plus (+) sign, and that all the leftward forces add with a minus (–) sign.
b. Don't forget about the x-components of any forces that are at an angle!
c. When you've added them all up, call this "the sum of all x forces" or "the net force in the x-direction."

6. *Add up all the* y-*forces and* y-*components.*

a. Remember that all the upward forces add with a (+) sign, all the downward forces add with a (–) sign.
b. Don't forget about the y-components of any forces that are at an angle!
c. When you've added them all up, call this "the sum of all y forces" or "net force in the y-direction"

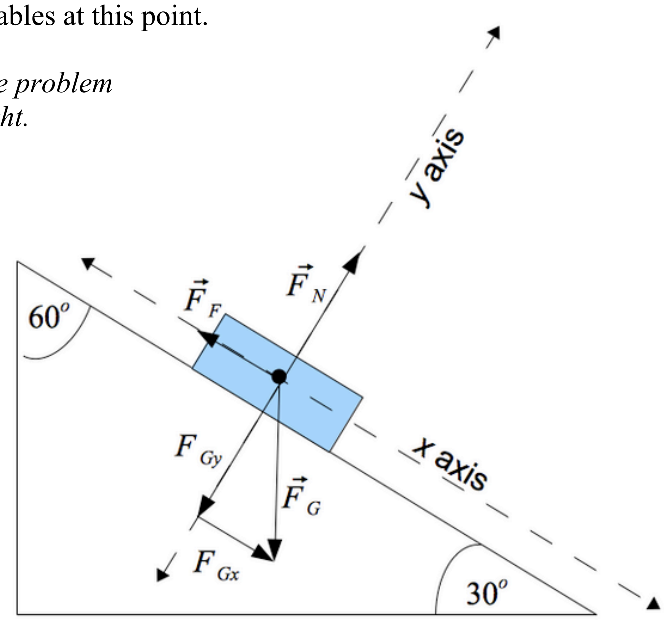
7. *Use Newton's Laws twice.*

a. The sum of all x-forces, divided by the mass, is the object's acceleration in the x-direction.
b. The sum of all y-forces, divided by the mass, is the object's acceleration in the y-direction.
c. If you happen to know that the acceleration in the x-direction or y-direction is zero (say the object is just sitting on a table), then you can plug this in to Newton's 2nd Law directly.
d. If you happen to know the acceleration, you can plug this in directly too.

8. *Each body should have a FBD.*

a. Draw a separate FBD for each body.
b. Set up a Σ f = ma equation based on the FBD for each body.
c. Newton's Third Law will tell you which forces on different bodies are the same in magnitude.
d. Your equations should equal your unknown variables at this point.

9. *For Incline planes it is usually easier to solve the problem if you rotate the x and y axis as shown to the right.*



Newton's Laws Problem Set

1. A VW Bug hits a huge truck head-on. Each vehicle was initially going 50 MPH.

    a. Which vehicle experiences the greater force?
    b. Which experiences the greater acceleration?

2. Is it possible for me to wave my hand and keep the rest of my body perfectly still? Why or why not?

3. How does a rocket accelerate in space, where there is nothing to 'push off' against?

4. Is there a net force on a hammer when you hold it steady above the ground? If you let the hammer drop, what's the net force on the hammer while it is falling to the ground?

5. If an object is moving at constant velocity or at rest, what is the minimum number of forces acting on it (other than zero)?

6. If an object is accelerating, what is the minimum number of forces acting on it?

7. You are standing on a bathroom scale. Can you reduce your weight by pulling up on your shoes? (Try it.)

8. When pulling a paper towel from a paper towel roll, why is a quick jerk more effective than a slow pull?

9. You and your friend are standing on identical skateboards with an industrial-strength compressed spring in between you. After the spring is released, it falls straight to the ground and the two of you fly apart.

    a. If you have identical masses, who travels farther?
    b. If your friend has a bigger mass who goes farther?
    c. If your friend has a bigger mass who feels the larger force?
    d. If you guys have identical masses, even if you push on the spring, why isn't it possible to go further than your friend?

10. Explain the normal force in terms of the microscopic forces between molecules in a surface.

11. A stone with a mass of 10 kg is sitting on the ground, not moving.

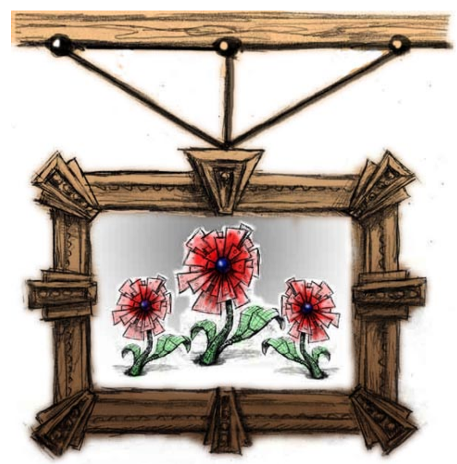
    a. What is the weight of the stone?
    b. What is the normal force acting on the stone?

12. The stone from the last question is now being pulled horizontally along the ground at constant speed in the positive x direction. Is there a net force on the stone?

13. A spring with spring constant $k = 400$ N/m has an uncompressed length of 0.23 m. When fully compressed, it has a length of 0.15 m. What force is required to fully compress the spring?

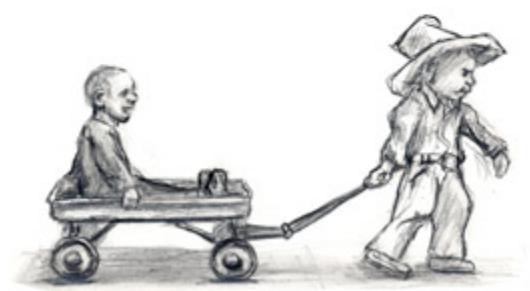
14. Measuring velocity is hard: for instance, can you tell how fast you're going around the Sun right now? Measuring acceleration is comparatively easy — you can *feel* accelerations. Here's a clever way to determine your acceleration. As you accelerate your car on a flat stretch, you notice that the fuzzy dice hanging from your rearview mirror are no longer hanging straight up and down. In fact, they are making a 30° angle with respect to the vertical. What is your acceleration? (Hint: Draw a FBD. Consider both x and y equations.)

15. Draw free body diagrams (FBDs) for all of the following objects involved (in **bold)** and label all the forces appropriately. Make sure the lengths of the vectors in your FBDs are proportional to the strength of the force: smaller forces get shorter arrows!



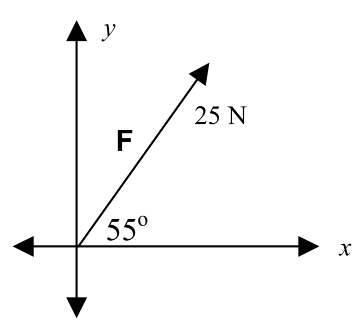
    a. A **man** stands in an elevator that is accelerating upward at 2 m/s^2.
    b. A boy is dragging a **sled** at a constant speed. The boy is pulling the sled with a rope at a 30° angle.
    c. Your **foot** presses against the ground as you walk.
    d. The **picture** shown here is attached to the ceiling by three wires.

16. Analyze the situation shown here with a big kid pulling a little kid in a wagon. You'll notice that there are a lot of different forces acting on the system. Let's think about what happens the moment the sled begins to move.

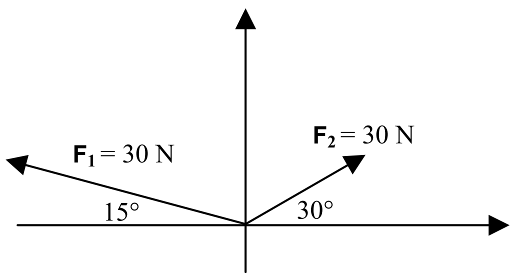


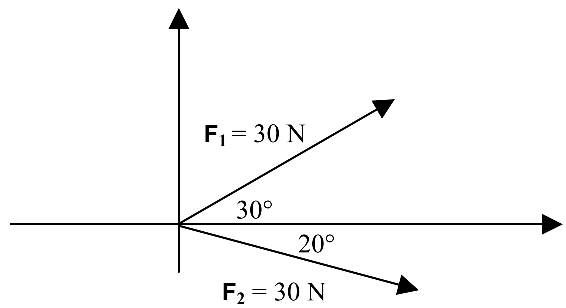
    a. First, draw the free body diagram of the big kid. Include all the forces you can think of, including friction. Then do the same for the little kid.
    b. Identify all third law pairs. Decide which forces act on the two body system and which are extraneous.
    c. Explain what conditions would make it possible for the two-body system to move forward.

17. Break the force vector **F** on the right into its *x* and *y* components, F_x and F_y.



18. For both figures below, find the net force and its direction (*i.e.*, the magnitude of $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$ and the angle it makes with the *x*-axis). Draw in **F**.

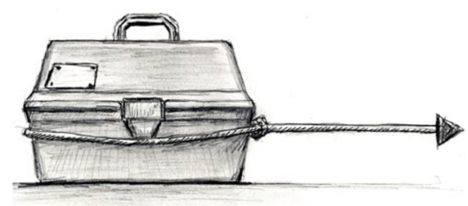




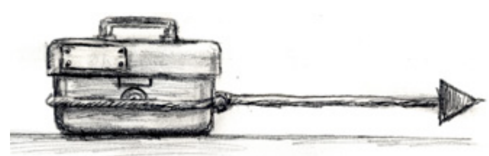
19. Andreas and Kaya are pulling a wagon. Andreas is pulling with a force of 50 N towards the northeast. Kaya is pulling with a force of 50 N towards the southeast. The wagon has a mass of 23 kg. What are the acceleration and direction of motion of the wagon?

20. Laura and Alan are pulling a wagon. Laura is pulling with a force of 50 N towards the northeast. Alan is pulling with a force of 50 N directly east. The wagon has a mass of 23 kg. What are the acceleration and direction of motion of the wagon?

21. When the 20 kg box to the right is pulled with a force of 100 N, it just starts to move (i.e. the maximum value of static friction is overcome with a force of 100 N). What is the value of the coefficient of static friction, μ_S?



22. A different box, this time 5 kg in mass, is being pulled with a force of 20 N and is sliding with an acceleration of 2 m/s^2. Find the coefficient of kinetic friction, μ_K.



23. The man is hanging from a rope wrapped around a pulley and attached to both of his shoulders. The pulley is fixed to the wall. The rope is designed to hold 500 N of weight; at higher tension, it will break. Let's say he has a mass of 80 kg. Draw a free body diagram and explain (using Newton's Laws) whether or not the rope will break.

24. Now the man ties one end of the rope to the ground and is held up by the other. Does the rope break in this situation? What precisely is the difference between this problem and the one before?

25. For a boy who weighs 500 N on Earth what are his mass and weight on the moon (where $g = 1.6\ m/s^2$)?

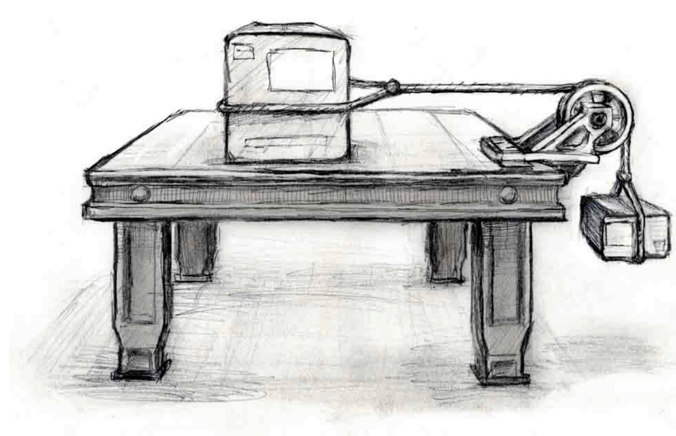
26. A woman of mass 70.0 kg weighs herself in an elevator.

    a. If she wants to weigh less, should she weigh herself when accelerating upward or downward?
    b. When the elevator is not accelerating, what does the scale read (i.e. what is the normal force that the scale exerts on the woman)?
    c. When the elevator is accelerating upward at 2.00 m/s^2, what does the scale read?

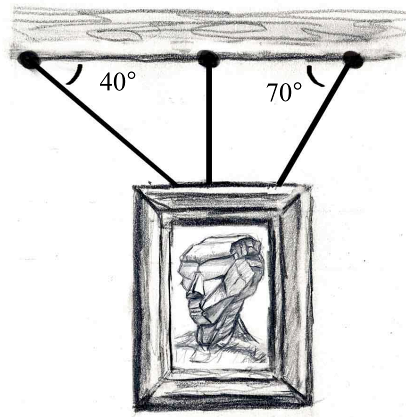
27. A crane is lowering a box of mass 50 kg with an acceleration of 2.0 m/s^2.

    a. Find the tension $\mathbf{F}_T$ in the cable.
    b. If the crane lowers the box at a constant speed, what is the tension $\mathbf{F}_T$ in the cable?

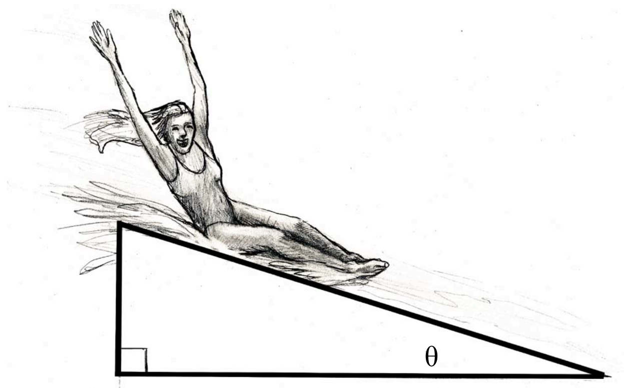
28. The large box on the table is 30 kg and is connected via a rope and pulley to a smaller 10 kg box, which is hanging. The 10 kg mass is the highest mass you can hang without moving the box on the table. Find the coefficient of static friction μ_S.



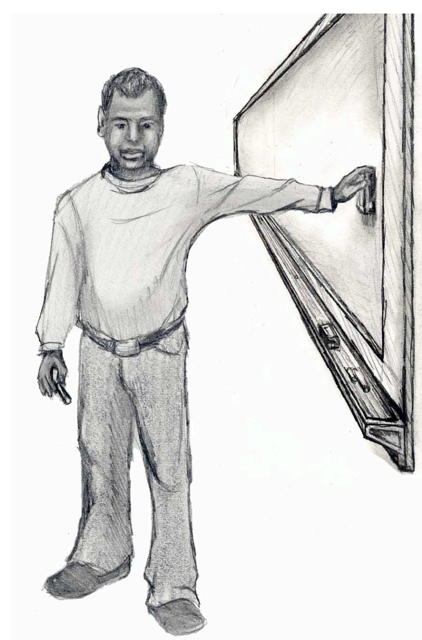
29. Find the mass of the painting. The tension in the leftmost rope is 7.2 N, in the middle rope it is 16 N, and in the rightmost rope it is 16 N.



30. Find Brittany's acceleration down the frictionless waterslide in terms of her mass m, the angle θ of the incline, and the acceleration of gravity g.



31. The physics professor holds an eraser up against a wall by pushing it directly against the wall with a completely horizontal force of 20 N. The eraser has a mass of 0.5 kg. The wall has coefficients of friction $\mu_S = 0.8$ and $\mu_K = 0.6$.

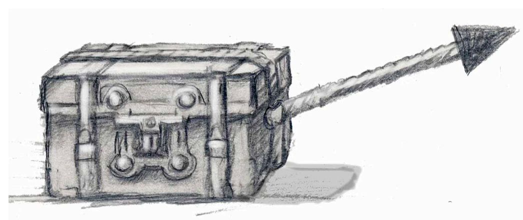


   a. Draw a free body diagram for the eraser.
   b. What is the normal force $\mathbf{F_N}$ acting on the eraser?
   c. What is the frictional force $\mathbf{F_S}$ equal to?
   d. What is the maximum mass m the eraser could have and still not fall down?
   e. What would happen if the wall and eraser were both frictionless?

32. A tractor of mass 580 kg accelerates up a 10° incline from rest to a speed of 10 m/s in 4 s. For all of answers below, provide a magnitude and a direction

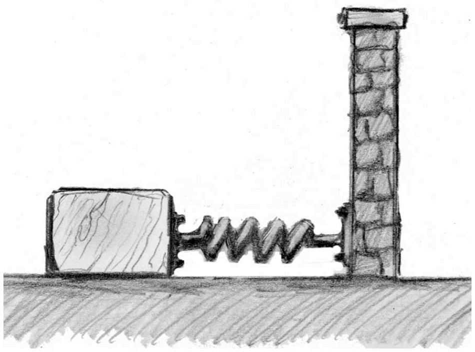
a. What net force $\mathbf{F}_{net}$ has been applied to the tractor?
b. What is the normal force, $\mathbf{F}_N$ on the tractor?
c. What is the force of gravity $\mathbf{F}_g$ on the tractor?
d. What force has been applied to the tractor so that it moves uphill?
e. What is the source of this force?

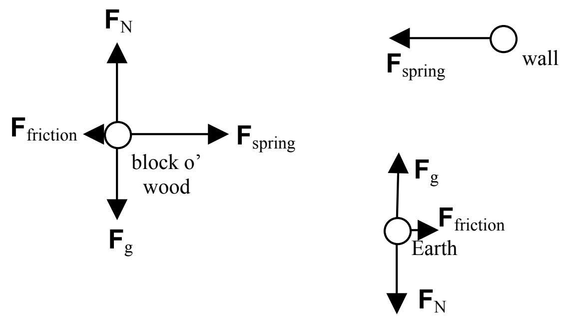
33. A heavy box (mass 25 kg) is dragged along the floor by a kid at a 30° angle to the horizontal with a force of 80 N (which is the maximum force the kid can apply).



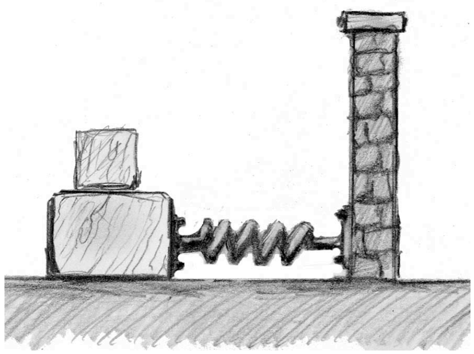
a. Draw the free body diagram.
b. What is the normal force $\mathbf{F}_N$?
c. Does the normal force decrease or increase as the angle of pull increases? Explain.
d. Assuming no friction, what is the acceleration of the box?
e. Assuming it begins at rest, what is its speed after ten seconds?
f. Is it possible for the kid to lift the box by pulling straight up on the rope?
g. In the absence of friction, what is the net force in the *x*-direction if the kid pulls at a 30° angle?
h. In the absence of friction, what is the net force in the *x*-direction if the kid pulls at a 45° angle?
i. In the absence of friction, what is the net force in the *x*-direction if the kid pulls at a 60° angle?
j. The kid pulls the box at constant velocity at an angle of 30°. What is the coefficient of kinetic friction μ_K between the box and the floor?
k. The kid pulls the box at an angle of 30°, producing an acceleration of 2 m/s^2. What is the coefficient of kinetic friction μ_K between the box and the floor?

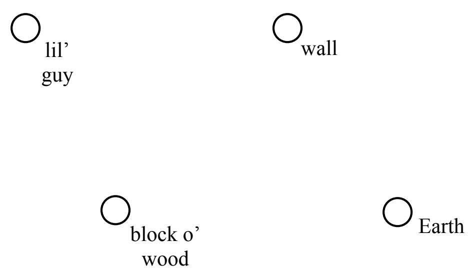
34. For the following situation, identify the 3rd law force pairs on the associated free body diagrams. Label each member of one pair "A," each member of the next pair "B," and so on. The spring is stretched so that it is pulling the block of wood to the right.



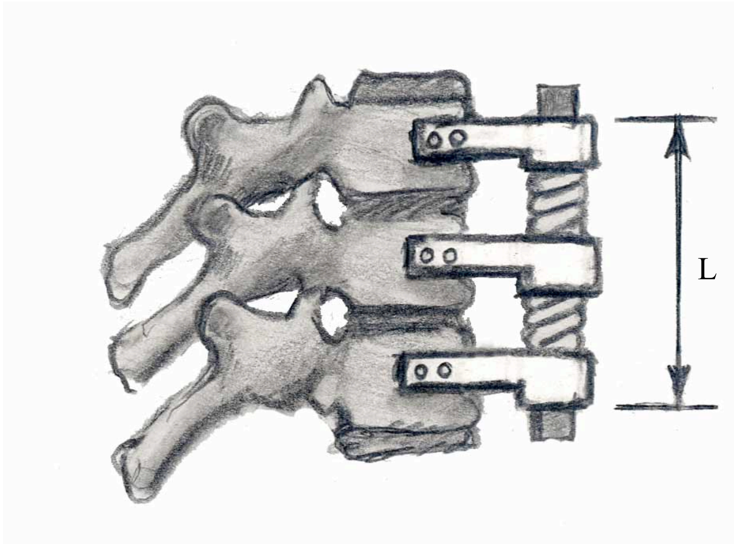


Draw free body diagrams for the situation below. Notice that we are pulling the bottom block *out from beneath* the top block. There is friction between the blocks! After you have drawn your FBDs, identify the 3rd law force pairs, as above.





35. Spinal implant problem — this is a real life bio-med engineering problem!



Here's the situation: both springs are compressed by an amount x_o. The rod of length L is fixed to both the top plate and the bottom plate. The two springs, each with spring constant k, are wrapped around the rod on both sides of the middle plate but are free to move, because they are not attached to the rod or the plates. The middle plate has negligible mass, and is constrained in its motion by the compression forces of the top and bottom springs.

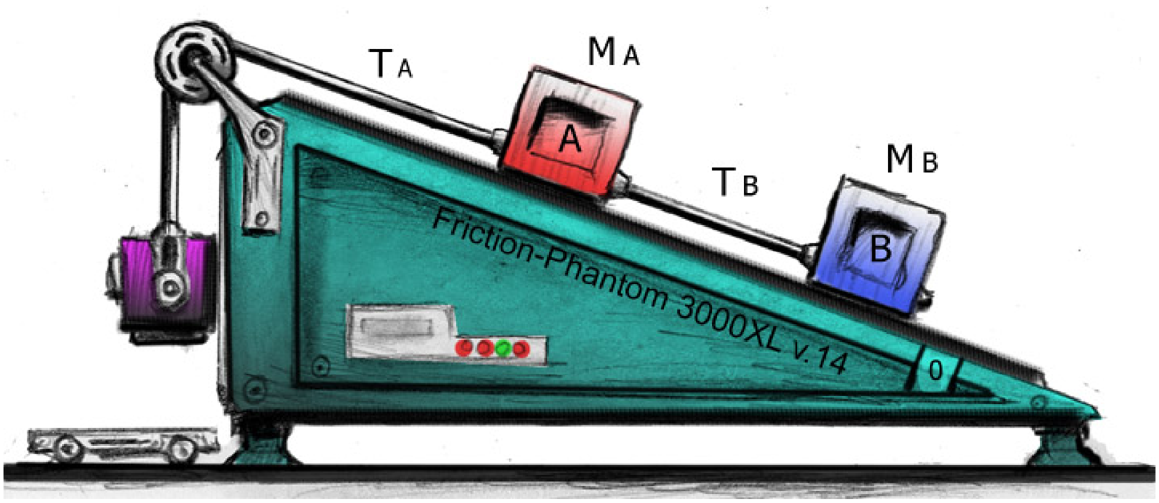
The medical implementation of this device is to screw the top plate to one vertebrae and the middle plate to the vertebrae directly below. The bottom plate is suspended in space. Instead of fusing broken vertebrates together, this implant allows movement somewhat analogous to the natural movement of functioning vertebrae. Below you will do the exact calculations that an engineer did to get this device patented and available for use at hospitals.

a. Find the force, **F**, on the middle plate for the region of its movement $\Delta x \le x_o$. Give your answer in terms of the constants given. (*Hint: In this region both springs are providing opposite compression forces.*)

b. Find the force, **F**, on the middle plate for the region of its movement $\Delta x \ge x_o$. Give your answer in terms of the constants given. (*Hint: In this region, only one spring is in contact with the middle plate.*)

c. Graph **F** vs. x. Label the values for force for the transition region in terms of the constants given.

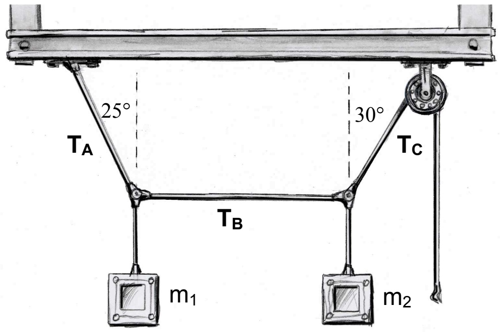
36. You design a mechanism for lifting boxes up an inclined plane by using a vertically hanging mass to pull them, as shown in the figure below.



The pulley at the top of the incline is massless and frictionless. The larger mass, M, is accelerating downward with a measured acceleration a. The smaller masses are m_A and m_B; the angle of the incline is θ, and the coefficient of kinetic friction between each of the masses and the incline has been measured and determined to be μ_K.

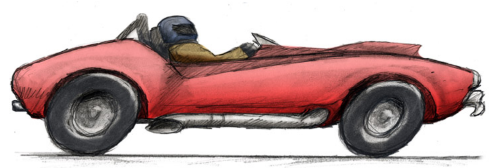
a. Draw free body diagrams for each of the three masses.
b. Calculate the magnitude of the frictional force on each of the smaller masses in terms of the given quantities.
c. Calculate the net force on the hanging mass in terms of the given quantities.
d. Calculate the magnitudes of the two tension forces T_A and T_B in terms of the given quantities.
e. Design and state a strategy for solving for how long it will take the larger mass to hit the ground, assuming at this moment it is at a height *h* above the ground. Do not attempt to solve this: simply state the strategy for solving it.

37. You build a device for lifting objects, as shown below. A rope is attached to the ceiling and two masses are allowed to hang from it. The end of the rope passes around a pulley (right) where you can pull it downward to lift the two objects upward. The angles of the ropes, measured with respect to the vertical, are shown. Assume the bodies are at rest initially.



a. Suppose you are able to measure the masses m_1 and m_2 of the two hanging objects as well as the tension T_C. Do you then have enough information to determine the other two tensions, T_A and T_B? Explain your reasoning.
b. If you only knew the tensions T_A and T_C, would you have enough information to determine the masses m_1 and m_2? If so, write m_1 and m_2 in terms of T_A and T_C. If not, what further information would you require?

38. A stunt driver is approaching a cliff at very high speed. Sensors in his car have measured the acceleration and velocity of the car, as well as all forces acting on it, for various times. The driver's motion can be broken down into the following steps:



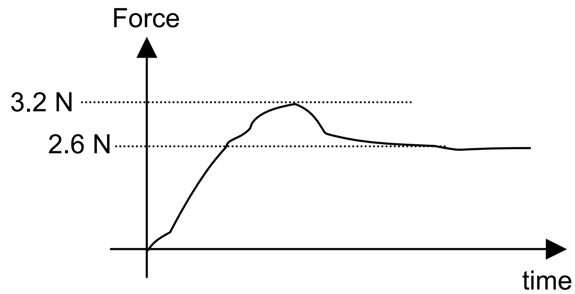
Step 1: The driver, beginning at rest, accelerates his car on a horizontal road for ten seconds. Sensors show that there is a force in the direction of motion of 6000 N, but additional forces acting in the opposite direction with magnitude 1000 N. The mass of the car is 1250 kg.

Step 2: Approaching the cliff, the driver takes his foot of the gas pedal (There is no further force in the direction of motion.) and brakes, increasing the force opposing motion from 1000 N to 2500 N. This continues for five seconds until he reaches the cliff.

Step 3: The driver flies off the cliff, which is 44.1 m high, and begins projectile motion.

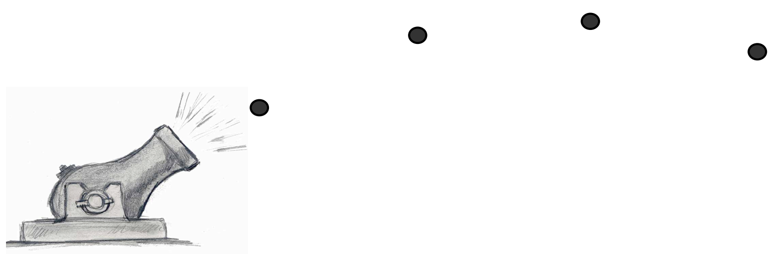
a. Ignoring air resistance, how long is the stunt driver in the air?
b. For Step 1:
   i. Draw a free body diagram, naming all the forces on the car.
   ii. Calculate the magnitude of the net force.
   iii. Find the change in velocity over the stated time period.
   iv. Make a graph of velocity in the *x*-direction vs. time over the stated time period.
   v. Calculate the distance the driver covered in the stated time period. Do this by finding the area under the curve in your graph of (iv). Then, check your result by using the equations for kinematics.
c. Repeat (b) for Step 2.
d. Calculate the distance that the stunt driver should land from the bottom of the cliff.

39. You are pulling open a stuck drawer with an electronic device that measures force. You measure the following behavior. The drawer has a weight of 7 N.



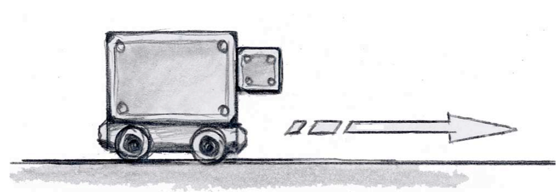
Draw a graph of friction force vs. time.

40. Draw arrows representing the **forces** acting on the cannonball as it flies through the air. Assume that air resistance is small compared to gravity, but not negligible.



41. A tug of war erupts between you and your sweetie. Assume your mass is 60 kg and the coefficient of friction between your feet and the ground is 0.5 (good shoes). Your sweetie's mass is 85 kg and the coefficient of friction between his/her feet and the ground is 0.35 (socks). Who is going to win? Explain, making use of a calculation.

42. A block has a little block hanging out to its side, as shown:



As you know, if the situation is left like this, the little block will just fall. But if we accelerate the leftmost block to the right, this will create a normal force between the little block and the big block, and if there is a coefficient of friction between them, then the little block won't slide down.

a. The mass of the little block is 0.15 kg. What frictional force is required to keep it from falling? (State a magnitude and direction.)
b. If both blocks are accelerating to the right with an acceleration **a** = 14.0 m/s^2, what is the normal force on the little block provided by the big block?
c. What is the minimum coefficient of static friction required?