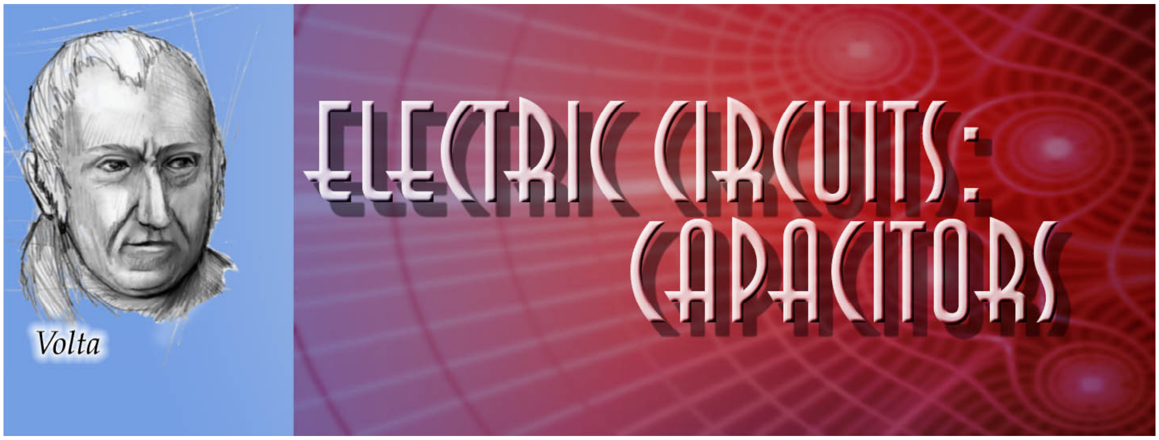


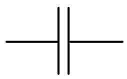
The Big Idea

When current flows through wires and resistors in a circuit as a result of an electric potential, charge does not build up significantly anywhere on the path. Capacitors are devices placed in electric circuits where charge *can* build up. The amount of charge a capacitor can store before it "fills up" depends on its shape and how much electric potential is applied. The larger the electric potential in Volts, the stronger the electric field that is used to "cram" the charge into the device. Eventually, all capacitors fill up when you put enough charge in them. This can be a way to store energy … by *discharging* the capacitor, you release the stored charge to flow and do your bidding.

Key Equations

- $Q = CV$ — the charge stored in a capacitor is equal to the capacitance multiplied by the electric potential. C is measured in Farads (F).
- $U = \frac{1}{2}CV^2$ — the potential energy stored in a capacitor is equal to one half the stored charge multiplied by the electric potential. U is measured in Joules.
- $C = \kappa\varepsilon_0 A/d$ — the capacitance of two parallel metal plates of area A, separated by distance d, and filled with a substance with dielectric constant κ is given by this formula. The permittivity, $\varepsilon_0 = 8.85\times10^{-12}\ C^2/Nm^2$, is a constant of nature.
- $Q(t) = Q_0\, e^{-t/\tau}$ — where $\tau = RC$. The amount of time, t, it takes to discharge about 2/3 of a capacitor is equal to the product of the resistor through which you are running current times the capacitance of the capacitor. To charge the capacitor the formula is $Q = Q_0\,(1-e^{-t/\tau})$.
- $I(t)=I_o\, e^{-t/\tau}$ — Formula for the current in the situation of charging or discharging a capacitor. It has the same exponential decay as a function of time.

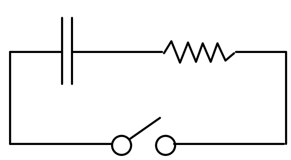
Electric Circuit Symbol



The symbol for a capacitor is two flat plates, mimicking the geometry of a capacitor, which typically consists of two flat plates separated by a small distance. The plates are normally wrapped around several times to form a cylindrical shape.

Key Concepts

- Current can flow *into* a capacitor from either side, but current doesn't flow across the capacitor from one plate to another. The plates do not touch, and the substance in between is insulating, not conducting.
- One side of the capacitor fills up with negative charge, the other with positive charge. The reason for the thin, close plates is you can use the negative charge on one plate to attract and hold the positive charge on the other plate. The reason for plates of big area is that you can spread out the charge on one plate so that its self-repulsion doesn't stop you from filling it with more charge.
- Typical dielectric constants κ are roughly 5.6 for glass and 80 for water. What these "dielectric" substances do is align their electric polarity with the electric field in a capacitor (much like atoms in a magnetic material) and, in doing so, reduces the electric field for a given amount of charge. Thereby allowing for more charge to be stored without repelling for a given Voltage.
- When a capacitor is initially uncharged, it is very easy to stuff charge in. As you put more charge in, it starts to build up and repel the additional charge you are attempting to stuff in there. The charging of a capacitor follows a logarithmic curve. The total amount of energy you need to expend to fully charge a capacitor with charge Q and electric potential V is $U = \frac{1}{2}QV = \frac{1}{2}CV^2$. When you pass current through a resistor into a capacitor, the capacitor eventually "fills up" and no more current flows. A typical RC circuit is shown below; when the switch is closed, the capacitor discharges with an exponentially decreasing current.



- Q refers to the amount of positive charge stored on the high voltage side of the capacitor; an equal and opposite amount, –Q, of negative charge is stored on the low voltage side of the capacitor.
- The total capacitance of two or more capacitors placed in series add as resistors *in parallel*: $1 / C_S = 1/C_1 + 1/C_{\ldots}$; two or more capacitors wired in parallel add as resistors *in series*: $C_P = C_1 + C_2 \ldots$.
- Many home-electronic circuits include capacitors; for this reason, it can be dangerous to mess around with old electronic components, as the capacitors may be charged even if the unit is unplugged. For example, old computer monitors (not flat screens) and TVs have capacitors that hold dangerous amounts of charge hours after the power is turned off.

Chapter 15: Electric Circuits – Capacitors Problems

1. Design a parallel plate capacitor with a capacitance of 100 mF. You can select any area, plate separation, and dielectric substance that you wish.

2. a. How much voltage would you have to apply to charge a 5 μF capacitor with 200 C of charge?
   b. Once you have finished, how much potential energy are you storing here?
   c. If all this energy could be harnessed to lift you up into the air, how high would you be lifted?

3. Show, by means of a sketch, showing the charge distribution, that two identical parallel-plate capacitors wired in parallel act exactly the same as a single capacitor with twice the area.

4. A certain capacitor can store 5 C of charge if you apply a voltage of 10 V.

   a. How many volts would you have to apply to store 50 C of charge in the same capacitor?
   b. Why is it harder to store more charge?

5. A certain capacitor can store 500 J of energy (by storing charge) if you apply a voltage of 15 V. How many volts would you have to apply to store 1000 J of energy in the same capacitor? (Important: why isn't the answer to this just 30 V?)

6. Marciel, a bicycling physicist, wishes to harvest some of the energy he puts into turning the pedals of his bike and store this energy in a capacitor. Then, when he stops at a stop light, the charge from this capacitor can flow out and run his bicycle headlight. He is able to generate 18 V of electric potential, on average, by pedaling (and using magnetic induction).

   a. If Mars wants to provide 0.5 A of current for 60 seconds at a stop light, how big a 18V capacitor should he buy (i.e. how many farads)?
   b. How big a resistor should he pass the current through so the RC time is three minutes?

7. Given a capacitor with 1 cm between the plates a field of 20,000 N/C is established between the plates.

   a. What is the voltage across the capacitor?
   b. If the charge on the plates is 1 μC what is the capacitance of the capacitor/
   c. If two identical capacitors of this capacitance are connected in series what it the total capacitance?
   d. Consider the capacitor connected in the following circuit at point B with two switches S and T, a 20 Ω resistor and a 120 V power source:
      i. Calculate the current through and the voltage across the resistor if S is open and T is closed
      ii. Repeat if S is closed and T is open

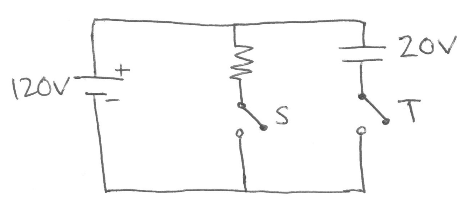
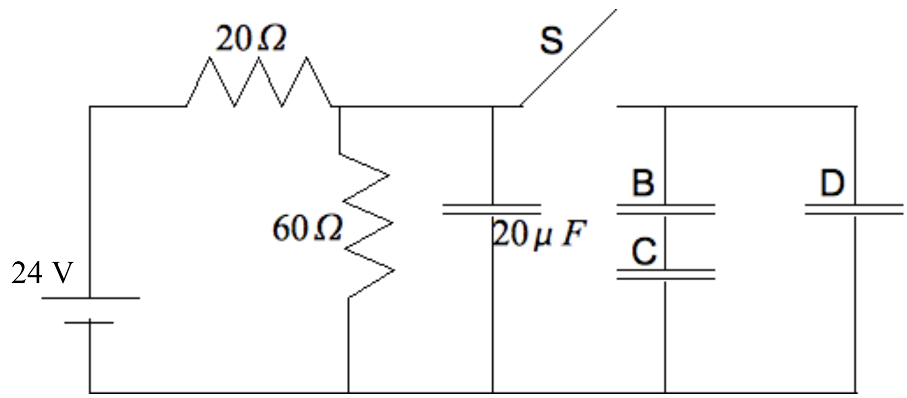


Figure for Problems 8-10:



8. Consider the figure above with switch, S, initially open:
   a. What is the voltage drop across the 20 Ω resistor
   b. What current flows thru the 60 Ωresistor
   c. What is the voltage drop across the 20 microfarad capacitor
   d. What is the charge on the capacitor
   e. How much energy is stored in that capacitor
   f. Find the capacitance of capacitors B, C, and D if compared to the 20 μF capacitor
      i. B has twice the plate area and half the plate separation
      ii. C has twice the plate area and the same plate separation
      iii. D has three times the plate area and half the plate separation
   g. Find the equivalent capacitance for the section after the switch (capacitors B,C,D).

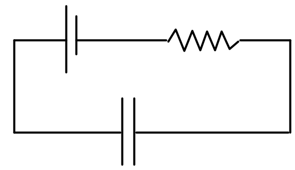
9. Now the switch in the previous problem is closed
   a. What is the total capacitance of branch II
   b. What is the total capacitance of branches I, II, III taken together
   c. What is the voltage drop across capacitor, B

10. Reopen the switch in the previous problem and look at the 20 μF capacitor. It has a plate separation of 2.0 mm.
    a. What is the magnitude and direction of the electric field
    b. If an electron is released in the center to traverse the capacitor of the capacitor and given a speed ⅔ the speed of light parallel to the plates what is the magnitude of the force on that electron
    c. What would be its acceleration in the direction perpendicular to its motion
    d. If the plates are 1.0 cm long how much time would it take to traverse the plate
    e. What displacement toward the plates would the electron undergo
    f. With what angle with respect to the direction of motion does the electron leave the plate

11. Design a circuit that uses capacitors, switches, voltage sources, and light bulbs that will allow the interior lights of your car to dim slowly once you get out.

12. Design a circuit that would allow you to determine the capacitance of an unknown capacitor.

13. The voltage source in the circuit below provides 10 V. The resistor is 200 Ω and the capacitor has a value of 50 μF. What is the voltage across the capacitor after the circuit has been hooked up for a long time?



14. A simple circuit consisting of a 39 μF and a 10 kΩ resistor. A switch is flipped connecting the circuit to a 12 V battery.

    a. How long until the capacitor has 2/3 of the total charge across it?
    b. How long until the capacitor has 99% of the total charge across it?
    c. What is the total charge possible on the capacitor?
    d. Will it ever reach the full charge in part c.?
    e. Derive the formula for V(t) across the capacitor.
    f. Draw the graph of V vs. t for the capacitor.
    g. Draw the graph of V vs. t for the resistor. }}

15. If you have a 39 μF capacitor and want a time constant of 5 seconds, what resistor value is needed?