The Big Idea

In studying fluids we apply the concepts of force, momentum, and energy, which we have learned previously, to new phenomena. Since fluids are made from a large number of individual molecules, we have to look at their behavior as a group. For this reason, we use the concept of conservation of *energy density* in place of conservation of energy. Energy density is energy divided by volume.

Key Concepts

- The *pressure* of a fluid is a measure of the forces exerted by a large number of molecules when they collide and bounce off a boundary. The unit of pressure is the Pascal (Pa).
- Mass density represents the amount of mass in a given volume. We also speak of fluids as having gravitational potential energy *density*, kinetic energy *density*, and momentum *density*. These represent the amount of energy or momentum possessed by a given volume of fluid.
- Pressure and energy density have the same units: 1 Pa = 1 N/m^2 = 1 J/m^3. The pressure of a fluid can be thought of as an arbitrary level of energy density.
- For static fluids and fluids flowing in a steady state all locations in the connected fluid system must have the same total energy density. This means that the algebraic sum of pressure, kinetic energy density and gravitational energy density equals zero. Changes in fluid pressure must be equal to changes in energy density (kinetic and/or gravitational).
- Liquids obey a *continuity equation* which is based on the fact that liquids are very difficult to compress. This means that the total volume of a sample of fluid will always be the same. Imagine trying to compress a filled water balloon …
- The *specific gravity* of an object is the ratio of the density of that object to the density of water. Objects with specific gravities greater than 1.0 (*i.e.*, greater than water) will sink in water; otherwise, they will float. The density of fresh water is 1000 kg/m^3.

Key Equations

- $P = F / A$; pressure is force per unit area.
- $\rho = M / V$; mass density ("rho") is mass over volume; the units of mass density are kg/m^3.
- $u_g = \rho g h$; gravitational potential energy density depends on the mass density of the fluid, the acceleration due to gravity, and the height of

the fluid above an (arbitrary) ground level.

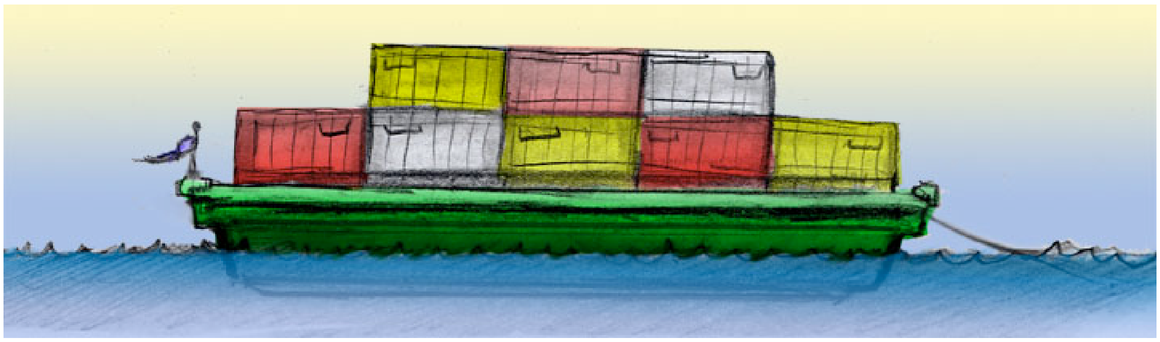
- $k = \frac{1}{2}\rho v^2$; kinetic energy density is related to the mass density and speed of the fluid.
- $P = \rho g h$; the pressure of a fluid depends on its depth .
- $\Delta P + \Delta k + \Delta u_g = 0$; a conserved quantity – *Bernoulli's Principle.*
- $\Phi = A \cdot v$; the flux of a fluid through a certain cross-sectional area depends on its speed.
- $F_{buoy} = \rho_{water}\, g V_{displaced}$; the buoyant force is equal to the weight of the displaced water.

Key Applications

- In a fluid at rest, pressure increases in proportion to its depth – this is due to the weight of the water above you.
- *Archimedes' Principle* states that the upward buoyant force on an object in the water is equal to the weight of the displaced volume of water. The reason for this upward force is that the bottom of the object is at lower depth, and therefore higher pressure, than the top. If an object has a higher density than the density of water, the weight of the displaced volume will be less than the object's weight, and the object will sink. Otherwise, the object will float.
- *Pascal's Principle* reminds us that, for a fluid of uniform pressure, the force exerted on a small area in contact with the fluid will be smaller than the force exerted on a large area. Thus, a small force applied to a small area in a fluid can create a large force on a larger area. This is the principle behind hydraulic machinery.
- *Bernoulli's Principle* is a restatement of the conservation of energy, but for fluids. The sum of pressure, kinetic energy density, and gravitational potential energy density is conserved. In other words, $\Delta P + \Delta k + \Delta u_g$ equals zero. One consequence of this is that a fluid moving at higher speed will exhibit a *lower* pressure, and vice versa. There are a number of common applications for this: when you turn on your shower, the moving water and air reduce the pressure in the shower stall, and the shower curtain is pulled inward; when a strong wind blows outside your house, the pressure decreases, and your shutters are blown open; due to the flaps on airplane wings, the speed of the air below the wing is lower than above the wing, which means the pressure below the wing is higher, and provides extra lift for the plane during landing. There are many more examples.
- Due to the conservation of flux, Φ, which means that a smaller fluid-carrying pipe requires a faster moving fluid, and also due to Bernoulli's Principle, which says that fast-moving fluids have low pressure, a useful rule emerges. Pressure in a smaller pipe must be lower than pressure in a larger pipe.
- If the fluid is not in a steady state energy can be lost in fluid flow. The loss of energy is related to *viscosity*, or deviation from smooth flow. Viscosity is related to *turbulence*, the tendency of fluids to become chaotic in their motion. In a high viscosity fluid, energy is lost from a fluid in a way that is quite analogous to energy loss due to current flow through a resistor. A pump can add energy to a fluid system also. The full Bernoulli Equation takes these two factors, viscosity and pumps, into account.

Fluids Problem Set

1. A block of wood with a density of 920 kg/m^3 is floating in a fluid of density 1100 kg/m^3. What fraction of the block is submerged, and what fraction is above the surface?



2. A rectangular barge 17 m long, 5 m wide, and 2.5 m in height is floating in a river. When the barge is empty, only 0.6 m is submerged. With its current load, however, the barge sinks so that 2.2 m is submerged. Calculate the mass of the load.

3. The density of ice is 90% that of water.

   a. Why does this fact make icebergs so dangerous?
   b. A form of the liquid naphthalene has a specific gravity of 1.58. What fraction of an ice cube would be submerged in a bath of naphthalene?

4. A cube of aluminum with a specific gravity of 2.70 and side length 4.00 cm is put into a beaker of methanol, which has a specific gravity of 0.791.

   a. Draw a free body diagram for the cube.
   b. Calculate the buoyant force acting on the cube.
   c. Calculate the acceleration of the cube toward the bottom when it is released.

5. A cube of aluminum (specific gravity of 2.70) and side length 4.00 cm is put in a beaker of liquid naphthalene (specific gravity of 1.58). When the cube is released, what is its acceleration?

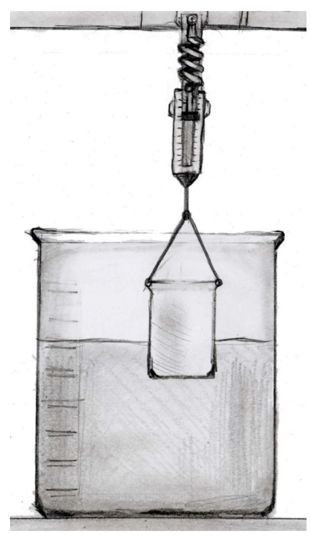
6. Your class is building boats out of aluminum foil. One group fashions a boat with a square 10 cm by 10 cm bottom and sides 1 cm high. They begin to put 2.5 g coins in the boat, adding them until it sinks. Assume they put the coins in evenly so the boat doesn't tip. How many coins can they put in? (You may ignore the mass of the aluminum boat … assume it is zero.)

7. You are riding a hot air balloon. The balloon is a sphere of radius 3.0 m and it is filled with hot air. The density of hot air depends on its temperature: assume that the density of the hot air is 0.925 kg/m^3, compared to the usual 1.29 kg/m^3 for air at room temperature. The balloon and its payload (including you) have a combined mass of 100 kg.

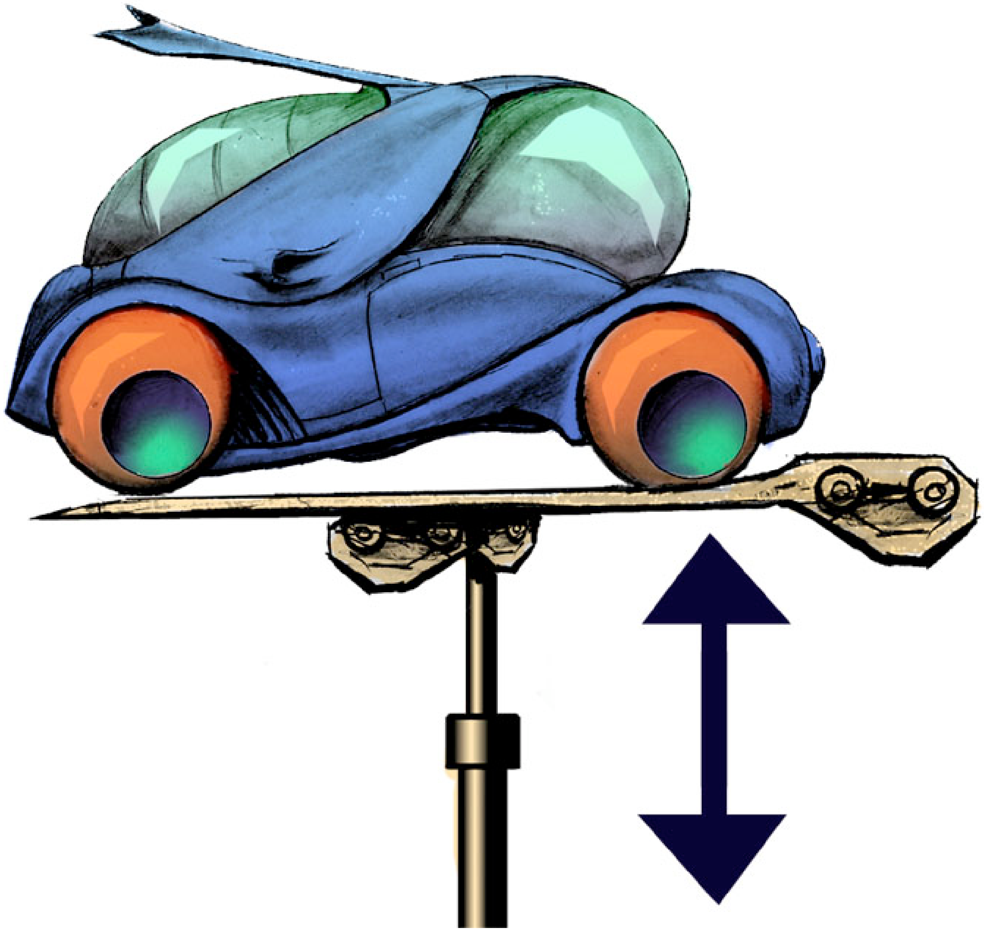
    a. Draw a free body diagram for the cube.
    b. Is the balloon accelerating upward or downward?
    c. What is the magnitude of the acceleration?
    d. Why do hot air ballooners prefer to lift off in the morning?
    e. What would limit the maximum height attainable by a hot air balloon?

8. You are doing an experiment in which you are slowly lowering a tall, empty cup into a beaker of water. The cup is held by a string attached to a spring scale that measures tension. You collect data on tension as a function of depth. The mass of the cup is 520 g, and it is long enough that it never fills with water during the experiment. The following table of data is collected:

String tension (N)	*Depth (cm)*	*Buoyant force (N)*
5.2	0	
4.9	1	
4.2	3	
3.7	5	
2.9	8	
2.3	10	
1.7	12	
0.7	15	
0.3	16	
0	17	



    a. Complete the chart by calculating the buoyant force acting on the cup at each depth.
    b. Make a graph of buoyant force vs. depth, find a best-fit line for the data points, and calculate its slope.
    c. What does this slope physically represent? (That is, what would a *greater* slope mean?)
    d. With this slope, and the value for the density of water, calculate the area of the circular cup's bottom and its radius.
    e. Design an experiment using this apparatus to measure the density of an unknown fluid.



9. A 1500 kg car is being lifted by a hydraulic jack attached to a flat plate. Underneath the plate is a pipe with radius 24 cm.

   a. If there is no net force on the car, calculate the pressure in the pipe.
   b. The other end of the pipe has a radius of 2.00 cm. How much force must be exerted at this end?
   c. To generate an upward acceleration for the car of 1.0 m/s^2, how much force must be applied to the small end of the pipe?

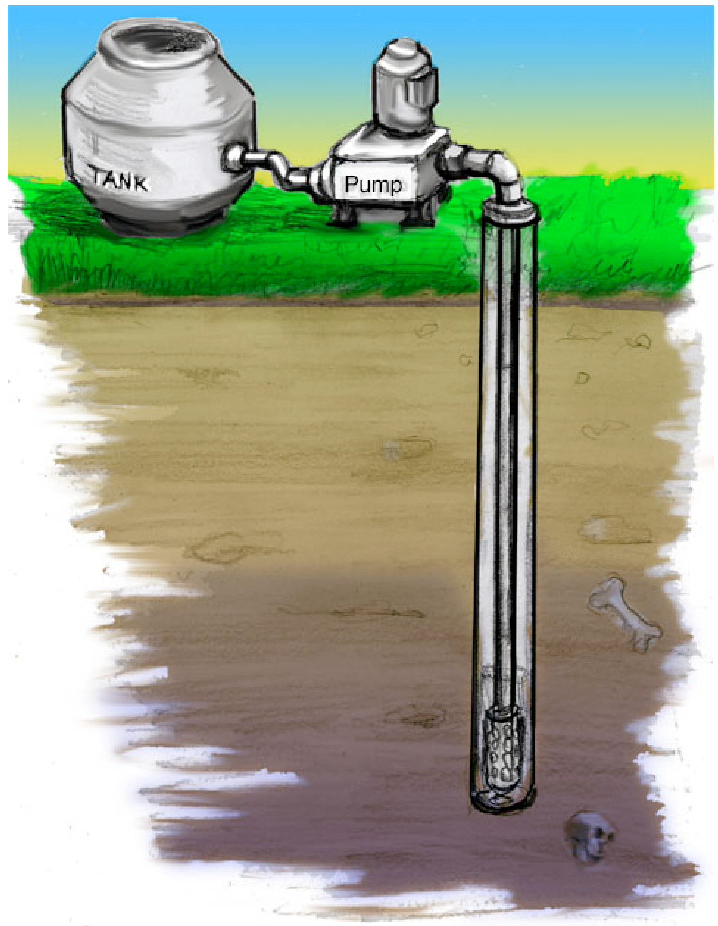
10. A SCUBA diver descends deep into the ocean. Calculate the water pressure at each of the following depths.

    a. 15 m
    b. 50 m
    c. 100 m

11. What happens to the gravitational potential energy density of water when it is siphoned out of a lower main ditch on your farm and put into a higher row ditch? How is this consistent with Bernoulli's principle?

12. Water flows through a horizontal water pipe 10.0 cm in diameter into a smaller 3.00 cm pipe. What is the ratio in water pressure between the larger and the smaller water pipes?

13. A pump is required to pipe water from a well 7.0 m in depth to an open-topped water tank at ground level. The pipe at the top of the pump, where the water pours into the water tank, is 2.00 cm in diameter. The water flow in the pipe is 5.00 m/s.



a. What is the kinetic energy density of the water flow?
b. What pressure is required at the bottom of the well? (Assume no energy is lost – i.e., that the fluid is traveling smoothly.)
c. What power is being delivered to the water by the pump?

(Hint: For the next part, refer to Chapter 12)

d. If the pump has an efficiency of 45%, what is the pump's electrical power consumption?
e. If the pump is operating on a 220 V power supply (typical for large pieces of equipment like this), how much electrical current does the pump draw?
f. At 13.5 cents per kilowatt-hour, how much does it cost to operate this pump for a month if it is running 5% of the time?

14. Ouch! You stepped on my foot! That is, you put a force of 550 N in an area of 9 cm^2 on the tops of my feet!

a. What was the pressure on my feet?
b. What is the ratio of this pressure to atmospheric pressure?

15. A submarine is moving directly upwards in the water at constant speed. The weight of the submarine is 500,000 N. The submarine's motors are off.

a. Draw a sketch of the situation and a free body diagram for the submarine.
b. What is the magnitude of the buoyant force acting on the submarine?

16. You dive into a deep pool in the river from a high cliff. When you hit the water, your speed was 20 m/s. About 0.75 seconds after hitting the water surface, you come to a stop before beginning to rise up towards the surface. Take your mass to be 60 kg.

    a. What was your average acceleration during this time period?
    b. What was the average net force acting on you during this time period?
    c. What was the buoyant force acting on you during this time period?

17. A glass of water with weight 10 N is sitting on a scale, which reads 10 N. An antique coin with weight 1 N is placed in the water. At first, the coin accelerates as it falls with an acceleration of $g/2$. About half-way down the glass, the coin reaches terminal velocity and continues at constant speed. Eventually, the coin rests on the bottom of the glass.

    What was the scale reading when…

    a. … the coin had not yet been released into the water?
    b. … the coin was first accelerating?
    c. … the coin reached terminal velocity?
    d. … the coin came to rest on the bottom?

18. You are planning a trip to the bottom of the Mariana Trench, located in the western Pacific Ocean. The trench has a maximum depth of 11,000 m, deeper than Mt. Everest is tall! You plan to use your bathysphere to descend to the bottom, and you want to make sure you design it to withstand the pressure. A bathysphere is a spherical capsule used for ocean descent – a cable is attached to the top, and this cable is attached to a winch on your boat on the surface.

    a. Name and sketch your bathysphere.
    b. What is the radius of your bathysphere in meters? (You choose – estimate from your picture.)
    c. What is the volume of your bathysphere in m^3?
    d. What is the pressure acting on your bathysphere at a depth of 11,000 m? The density of sea water is 1027 kg/m^3.
    e. If you had a circular porthole of radius 0.10 m (10 cm) on your bathysphere, what would the inward force on the porthole be?
    f. If the density of your bathysphere is 1400 kg/m^3, what is the magnitude of the buoyant force acting on it when it is at the deepest point in the trench?
    g. In order to stop at this depth, what must the tension in the cable be? (Draw an FBD!)