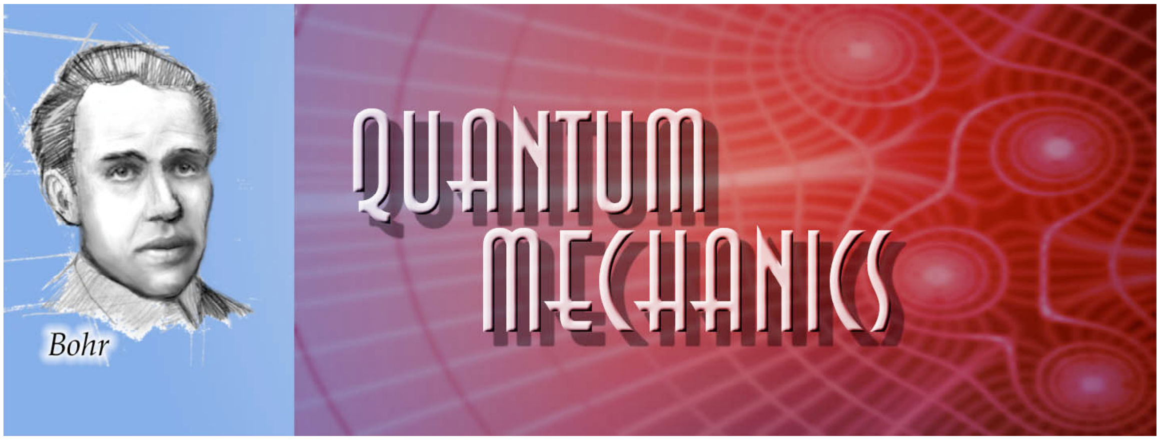


The Big Idea

Quantum Mechanics, discovered early in the 20th century, completely shook the way physicists think. Quantum Mechanics is the description of how the universe works on the very small scale. It turns out that we can predict not what will happen, but only the probability of what will happen. The uncertainty of quantum events is extremely important at the atomic or smaller level but not important at the macroscopic level. In fact the correspondence principle states that all results from quantum mechanics must agree with classical physics as the mass increases. The foundation of quantum mechanics was developed on the observation of *wave-particle duality.*

Electromagnetic Radiation is carried by particles, called photons, which interact with electrons. Depending on the experiment photons can behave as particles or waves. The reverse is also true; electrons can also behave as particles or waves.

Because the electron has a wavelength its position and momentum can never be precisely established. This is called the uncertainty principle. (What has been said above about the electron is true for protons or any other particle, but, experimentally, the effects become undetectable with increasing mass.)

The Key Concepts

- The energy of a photon is the product of the frequency and Planck's Constant. This is the exact amount of energy an electron will have if it absorbs a photon.
- A photon, which has neither mass nor volume, carries energy and momentum; the quantity of either energy or momentum in a photon depends on its frequency. The photon travels at the speed of light.
- If an electron loses energy the photon emitted, to obey the law of energy conservation, will have its frequency (and wavelength) determined by the electron's energy..
- An electron, which has mass (but probably no volume) has energy and momentum determined by its speed, which is always less than that of light. The electron has a wavelength determined by its momentum.
- If a photon strikes some *photoelectric* material its energy must first go into releasing the electron from the material (This is called the work function of the material.) The remaining energy, if any, goes into kinetic energy of the electron and the voltage of an electric circuit can be calculated from this. The current comes from the number of electrons/second and that corresponds exactly to the number of photons/second.

- Increasing the number of photons will not change the amount of energy an electron will have, but will increase the number of electrons emitted
- The momentum of photons is equal to Planck's constant divided by the wavelength
- The wavelength of electrons is equal to Planck's constant divided by the electron's momentum. If an electron is traveling at about .1c this wavelength is then not much smaller than the size of an atom.
- The size of the electron's wavelength determines the possible energy levels in an atom. These are negative energies since the electron is said to have zero potential energy when it is ionized. The lowest energy level (ground state) for hydrogen is -13.6 ev. The second level is -3.4 ev. Atoms with multiple electrons have multiple sets of energy levels. (And energy levels are different for partially ionized atoms.)
- When an electron absorbs a photon it moves to higher energy level, depending on the energy of the photon. If a 13.6 ev photon hits a hydrogen atom it ionizes that atom. If a 10.2 ev photon strikes hydrogen the electron is moved to the next level.
- Atomic spectra are unique to each element. They are seen when electrons drop from a higher energy level to a lower one. For example when an electron drops from -3.4 ev to -13.6 ev in Hydrogen then a 10.2 ev photon is emitted. The spectra can be in infra-red, visible light, ultra-violet and even X-rays. (The 10.2 ev photon is ultra-violet.)
- The wave nature of electrons makes it impossible to determine exactly both its momentum and position. The product of the two uncertainties is of the order of Plank's Constant. (Uncertainty in the electron's energy and time are likewise so related.)

The Key Equations

$E = hf$; Relates energy of a photon to its frequency.

$p = h/\lambda$; Relates the momentum of a photon to its wavelength.

$\lambda = h/p$; The Debroglie wavelength of an electron.

$(\Delta x)(\Delta p) \geq h/4\pi$; This is the Heisenberg Uncertainty Principle, (HUP) and relates the minimum measurable quantities in momentum and position of a particle.

$(\Delta E)(\Delta t) \geq h/4\pi$; Relates the uncertainty in measuring the energy of a particle and the time it takes to do the measurement.

$h = 6.626 \times 10^{-34}$ J-sec ; Planck's constant.

$1 \text{ ev} = 1.602 \times 10^{-19}$ J ; The most convenient unit of energy at the atomic scale is the electonvolt, defined as the potential energy of the charge of an electron across a potential difference of 1 volt.

1240nm ~ 1ev ; A photon of energy of 1.00 ev has a wavelength of 1240 nm and vice versa. This is a convenient shortcut for determining the wavelengths of photons emitted when electrons change energy levels, or for calculations involving the photoelectric effect.

Problems Set: Quantum Mechanics

1. Calculate the energy and momentum of photons with the following frequency;

   a. From an FM station at 101.9 MHz
   b. Infrared radiation at 0.90 x 10^{14} Hz
   c. From an AM station at 740 kHz

2. Find the energy and momentum of photons with a wavelength:

   a. red light at 640 nm
   b. ultraviolet light at 98.0 nm
   c. gamma rays at .248 pm

3. Given the energy of the following particles find the wavelength of:

   a. X-ray photons at 15.0 kev
   b. Gamma ray photons from sodium 24 at 2.70 Mev
   c. A 1.70 ev electron

4. The momentum of an electron is measured to an accuracy of 5.10 x10^{-15} kg-m/s. What is the corresponding uncertainty in the position of the electron?

5. The four lowest energy levels in electron-volts in a hypothetical atom are respectively: -34 ev, -17 ev, -3.5 ev, -.27 ev.

   a. Find the wavelength of the photon that can ionize this atom.
   b. Is this visible light? Why?
   c. If an electron is excited to the fourth level what are the wavelengths of all possible transitions? Which are visible?

6. Light with a wavelength of 620 nm strikes a photoelectric surface with a work function of 1.20 ev. What is the stopping potential for the electron?

7. For the same surface in problem 7 but different frequency light a stopping potential of 1.40 V is observed. What is the wavelength of the light?

8. An electron is accelerated through 5000 V. It collides with a positron of the same energy. All energy goes to produce a gamma ray.

   a. What is the wavelength of the gamma ray ignoring the rest mass of the electron and positron?
   b. Now calculate the contribution to the wavelength of the gamma ray of the masses of the particles? Recalculate the wavelength.
   c. Was it safe to ignore their masses? Why or why not?

9. An photon of 42.0 ev strikes an electron. What is the increase in speed of the electron assuming all the photon's momentum goes to the electron?

10. A 22.0 kev X-ray in the x-direction strikes an electron initially at rest. This time a 0.1 nm X-ray is observed moving in the x-direction after collision. What is the magnitude and direction of the velocity of the electron after collision?

11. The highly radioactive isotope Polonium 214 has a half-life of 163.7 μs and emits a 799 kev gamma ray upon decay. The isotopic mass is 213.99 amu.

a. How much time would it take for 7/8 of this substance to decay?
b. Suppose you had 1.00 g of Po^{214} how much energy would the emitted gamma rays give off while 7/8 decayed?
c. What is the power generated in kilowatts?
d. What is the wavelength of the gamma ray?

12. Ultra-violet light of 110 nm strikes a photoelectric surface and requires a stopping potential of 8.00 volts. What is the work function of the surface?

13. Students doing an experiment to determine the value of Planck's constant shined light from a variety of lasers on a photoelectric surface with an unknown work function and measured the stopping voltage. Their data is summarized below:

laser	Wavelength (nm)	Voltage (V)
Helium-Neon	632.5	.50
Krypton-Flouride	248	3.5
Argon	488	1.1
Europium	612	.60
Gallium arsenide	820	.05

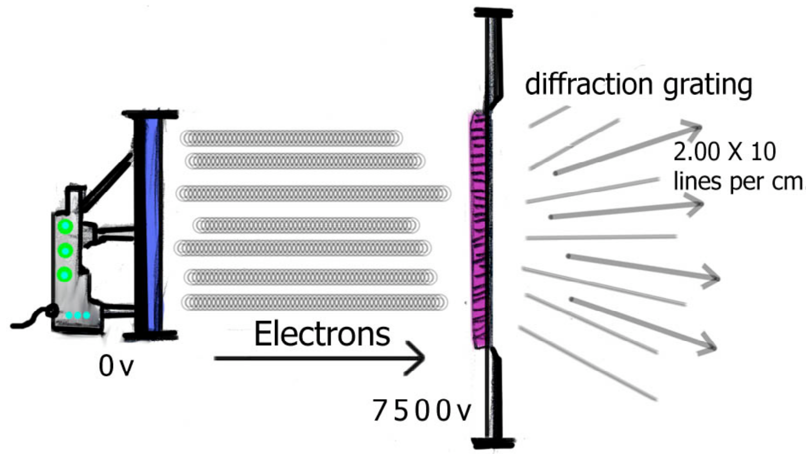
a. Construct a graph of energy vs. frequency of emitted electrons.
b. Use the graph to determine the *experimental value of* Planck's constant
c. Use the graph to determine the work function of the surface
d. Use the graph to determine what wavelength of light would require a 6.0 V stopping potential.
e. Use the graph to determine the stopping potential required if 550 nm light were shined on the surface.

14. An element has the following six lowest energy (in ev) levels for its outermost electron: -24 ev, -7.5 ev, -2.1 ev, -1.5 ev. -.92 ev, -.69 ev.

a. Construct a diagram showing the energy levels for this situation.
b. Show all possible transitions; how many are there?
c. Calculate the wavelengths for transitions to the -7.5 ev level
d. Arrange these to predict which would be seen by infrared, visible and ultraviolet spectroscopes

15. A different element has black absorption lines at 128 nm, 325 nm, 541 nm and 677 nm when white light is shined upon it. Use this information to construct an energy level diagram.

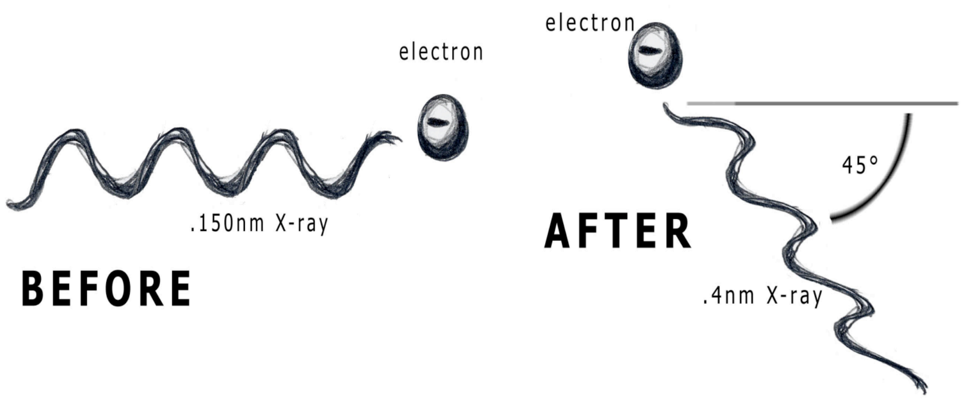
16. An electron is accelerated through 7500 V and is beamed through a diffraction grating, which has 2.00×10^7 lines per cm.



a. Calculate the speed of the electron
b. Calculate the wavelength of the electron
c. Calculate the angle in which the first order maximum makes with the diffraction grating
d. If the screen is 2.00 m away from the diffraction grating what is the separation distance of the central maximum to the first order?

17. A light source of 429 nm is used to power a photovoltaic cell with a work function of 0.900 ev. The cell is struck by 1.00×10^{19} photons per second.

a. What voltage is produced by the cell?
b. What current is produced by the cell?
c. What is the cell's internal resistance?



18. A .150 nm X-ray moving in the positive x-direction strikes an electron, which is at rest. After the collision an X-ray of 0 .400 nm is observed to move 45 degrees from the positive x-axis.

a. What is the initial momentum of the incident X-ray?
b. What are the x and y components of the secondary X-ray?
c. What must be the x and y components of the electron after collision?
d. Give the magnitude and direction of the electrons' final velocity.

19. Curium 242 has an isotopic mass of 242.058831 amu and decays by alpha emission; the alpha particle has a mass of 4.002602 amu and has a kinetic energy of 6.1127 Mev.

a. What is the momentum of the alpha particle?
b. What is its wavelength?

c. Write a balanced nuclear equation for the reaction.
d. Calculate the isotopic mass of the product.
e. If the alpha particle is placed in a magnetic field of .002 T what is the radius of curvature? (The alpha particle has a double positive charge.)
f. If the alpha particle is moving in the x-direction and the field is in the z-direction find the direction of the magnetic force.
g. Calculate the magnitude and direction of the electric field necessary to make the alpha particle move in a straight line.

20. A student lab group has a laser of unknown wavelength, a laser of known wavelength, a photoelectric cell of unknown work function, a voltmeter and test leads, and access to a supply of resistors.

a. Design an experiment to measure the work function of the cell, and the wavelength of the unknown laser. Give a complete procedure and draw an appropriate circuit diagram. Give sample equations and graphs if necessary.
b. Under what circumstances would it be impossible to measure the wavelength of the unknown laser?
c. How could one using this apparatus also measure the intensity of the laser (number of photons emitted/second)?

21. The momentum of an electron is measured to an accuracy of $\pm 5.1 \times 10^{-24}$ kg·m/s. What is the corresponding uncertainty in the position of the same electron at the same moment? Express your answer in Angstroms (1 Å = 10^{-10} m, about the size of a typical atom).

22. Thor, a baseball player, passes on a pitch clocked at a speed of 45 ± 2 m/s. The umpire calls a strike, but Thor claims that the uncertainty in the position of the baseball was so high that Heisenberg's uncertainty principle dictates the ball *could* have been out of the strike zone. What is the uncertainty in position for this baseball? A typical baseball has a mass of 0.15 kg. Should the umpire rethink his decision?

23. Consider a box of empty space (vacuum) that contains nothing, and has total energy $E = 0$. Suddenly, in seeming violation of the law of conservation of energy, an electron and a positron (the anti-particle of the electron) burst into existence. Both the electron and positron have the same mass, 9.11×10^{-31} kg.

a. Use Einstein's formula ($E = mc^2$) to determine how much energy must be used to create these two particles out of *nothing*.
b. You don't get to violate the law of conservation of energy forever – you can only do so as long as the violation is "hidden" within the HUP. Use the HUP to calculate how long (in seconds) the two particles can exist before they wink out of existence.
c. Now let's assume they are both traveling at a speed of 0.1 c. (Do a non-relativistic calculation.) How far can they travel in that time? How does this distance compare to the size of an atom?
d. What if, instead of an electron and a positron pair, you got a proton/anti-proton pair? The mass of a proton is about 2000× higher than the mass of an electron. Will your proton/anti-proton pair last a *longer* or *shorter* amount of time than the electron/positron pair? Why?